From Conservative to Dissipative：

Dynamics and Variational Method of Quasi-Hamiltonian Systems

北京師範大學

中國科學院
CHINESE ACADEMY OF SCIENCES

Dynamic Online Forums

As the front line of exploring the physical world，Hamilton dynamic system always stands in the stage center of the research on differential equations. Since the Age of Newton, each technological revolution is supported by great breakthroughs of this subject, with no exception. Since 1980s, the variational approach was involved and individually developed, more complicated dynamic phenomena for high dimensional systems was explosively revealed and verified.

In fact, Hamiltonian systems with damping are preciser in describing the realistic physical motions. The presence of stable attractors dominate the global dynamics of the equations. From the viewpoint of variational method, such a kind of systems has an apparent optimal control meaning, which brings us with exuberant variational properties. Moreover, the application on the 1st order PDE drawn the attention of a bunch of famous mathematicians including Arnold, Lions, etc.

This series of reports aims to explore the topological properties and geometric rigidity of quasi-Hamilton systems. We also try to exhibit the research perspective of it on modern physics、astronomy、economics, and even numerical computation、machine learnings.

Time: 2020/7/1-2020/12/31

Organization： Academy of Mathematics and systems science, Chinese Academy of Sciences, Beijing Normal University

Committee: Prof. C-Q Cheng (Nanjing University)

Prof. Zaijiu Shang (AMSS,CAS)

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Organizer: Xifeng Su (Beijing Normal University)

Jianlu Zhang （AMSS, CAS）

Date: July 10, 2020
Time: 14:00- 16:00 (Beijing Time)
Tencent Meeting ID: 242 605 286
Password: 123456

Speaker 1: Jianlu Zhang (Institute of Mathematics, CAS)
Title: Global dynamics of the symplectic twist mappings
Abstract: As an object of low-dimensional structure-preserving dynamical systems, symplectic twist mappings become the most thoroughly studied topic in Hamiltonian dynamics because of their wide applications and nice topological properties. The introduction of variational methods also provides numerous intuitive bases for exploring the higher-dimensional dynamics. We will focus on introducing the global parametrization of weak KAM solutions in order to interpret the mechanisms of how the singularities and local transition chains appear.

Speaker 2: Liang Jin (Nanjing University of Science and Technology)
Title: Global dynamics of the characteristic systems for the first order partial differential equations
Abstract: The characteristic systems associated to the first order partial differential equations is an important extension of the classical Hamiltonian systems. In this talk, we will introduce the development of such systems and interpret our recent work on the maximal global attractor (repellor) and global dynamics of these strictly dissipative (stimulating type) characteristic systems. Our results show that a large class of systems with dissipative or stimulating mechanisms could be included in the research framework of the characteristic systems herein.

Date: July 17, 2020
Time: 14:00- 16:00 (Beijing Time)
ZOOM ID: 690 776 66313
Password: 123456

Speaker 1: Lin Wang (Yau Mathematical Sciences Center)
Title: Contact Hamiltonian systems from the variational viewpoint
Abstract: Based on the Aubry-Mather theory and weak KAM theory in classical Hamiltonian systems, we establish and develop the global action minimizing methods for contact Hamiltonian systems. In this talk, I will introduce the fundamental tools of these methods and the new phenomena of the contact Hamiltonian systems (discovered by these tools), which are quite different from those of classical Hamiltonian systems. This talk is based on a series of joint works with Xifeng Su, Kaizhi Wang and Jun Yan.

Speaker 2: Kai Zhao (Fudan University)
Title: Convergence rate estimates of viscosity solutions of linearly damping Hamiltonian systems
Abstract: Linearly damping Hamiltonian systems, which are special characteristic systems, have wide applications into tidal traction in celestial mechanics, physical models of resistance effect and economic decay effect. Due to the speciality of the first order partial differential equations, experts are concerned with the convergence rate estimates of the associated viscosity solutions in order to obtain the dynamical information of the Aubry sets with variational properties. We will concentrate on the analysis of convergence rate for the two most commonly concerned forms of Aubry sets (KAM tori and hyperbolic periodic orbits).

Date: July 24, 2020
Time: 14:00- 16:00 (Beijing Time)
ZOOM ID: 661 689 45483
Password: 123456

Speaker 1: Jun Yan (Fudan University)
Title: Hamilton-Jacobi: from classic to contact
Abstract: We introduce the Hamilton-Jacobi methods in classical Hamiltonian systems and contact Hamiltonian systems and discuss about the applications of classical and viscosity solutions of Hamilton-Jacobi equations in dynamical systems.

Speaker 2: Ya-nan Wang (Nanjing Normal University)
Title: ordered structures in Frenkel-Kontorova models
Abstract: Frenkel-Kontorova models (FK models) describe a motion law of a series of interaction particles subjected to given potents. The ordered structures is crucial for the stability of the systems. In this talk, we will mainly talk about the properties of ordered structures in damped FK models, including the criteria for the existence of the leaf-like structures, the properties of foliated structures and so on.

Date: July 31, 2020
Time: 14:00- 16:00 (Beijing Time)
Tencent Meeting ID: 826 339 058
Password: 123456

Speaker 1: Tianhong Li (Institute of Mathematics, CAS)
Title: Global structure and regularity of solutions of Hamilton-Jacobi equations
Abstract: Hamilton-Jacobi equations are first order partial differential equations and their characteristic equations are Hamiltonian ordinary differential equations. We study the topological properties and global structures of the singular sets of solutions of Hamilton-Jacobi equations, and the regularity of solutions outside the singular sets. This talk is based on a series of joint works with Jinghua Wang and Hairui, Wen.

Speaker 2: Wei Cheng (Nanjing University)
Title: Preview of the applications of singularities from H-J equations
Abstract: We try to review known and unknown results on singularities and their propagation from Hamilton-Jacobi equations in the past decade. We will emphasize the potential problem and observation for the applications to dynamical systems, geometry, calculus of variation and optimal control and PDE. This talk can be regarded as an extension of the preprint "On and beyond propagation of singularities of viscosity solutions", arXiv:1805.11583, 2018, by Cannarsa and the speaker.

Date: August 7, 2020
Time: 14:00- 16:00 (Beijing Time)
Tencent Meeting ID: 581 311 991
Password: 123456

Speaker 1: Yiqian Wang (Nanjing University)
Title: On quasi-periodic Schrodinger operators with cos-type potentials
Abstract: Quasiperiodic Schrodinger operators (QPSO) is the mathematical model for the conductivity on quasi-crystals which was found by a Nobel prize winner. Several great mathematicians have been captivated by this field. In last decades, various methods have been developed in the study of one-dimensional analytic QPSO, which led to a lot of deep results. However, these methods depend heavily on analytic conditions and are difficult to be extended to smooth situations. Recently, a series of sharp results for Sinai's model (QPSO with a C^2 cos-type potential and a large coupling) have been obtained. In this talk, we will discuss the roles played by geometric conditions and regularity conditions on the potentials as well as the relationship between them.

Speaker 2: Qiaoling Wei (Capital Normal University)
Title: Can you hear the shape of a drum: inverse spectral problem of convex planar domains
Abstract: The famous question "Can you hear the shape of drum?" Is raised by M.Kac in 1960. The corresponding inverse spectral problem in Mathematics is: whether a planar domain Ω can be uniquely determined by its Laplace spectrum consisting of eigenvalues of a Dirichlet problem. On the other hand, some dynamical systems (billiard systems) could be given on Ω, and the length spectrum consists of perimeters of all periodic orbits of a billiard problem inside Ω. The Laplace and length spectra are closely related. We will introduce the history and the recent development of inverse spectral problem, and mainly concerns the related approaches of the length spectra of billiard systems.

Date: August 14, 2020
Time: 14:00- 16:00 (Beijing Time)
Tencent Meeting ID: 384 498 320
Password: 123456

Speaker 1: Xiaojun Cui (Nanjing University)
Title: Aubry-Mather theory and weak KAM theory in geometrical framework
Abstract: We will mainly review and summarize the main results of Aubry-Mather theory and weak KAM theory in geometric (Riemann and Lorentz) framework, and discuss some relations between these theories and geometric measure theory, rigidity, geo-dynamics of Wasserstein space, etc.

Speaker 2: Lu Xu (Jilin University)
Title: Poincare Mechanism in Multi-scaled Hamiltonian Systems
Abstract: My talk is about the quasi-periodic motions in multi-scaled Hamiltonian systems. It consists of four part. At first, I will introduce the results in integrable Hamiltonian systems since what we focus on is nearly-integrable Hamiltonian system. The second part is the definition of nearly-integrable Hamiltonian system and the classical KAM theorem. After then, I will introduce that what is Poincare problem and some interesting results corresponding to this problem. The last part, which is also the main part, I will talk about the definition and the back ground of nearly- integrable Hamiltonian system, then the persistence of lower dimensional tori on resonant surface, which is our recent result. I will also simply introduce the Technical ingredients of our work.

Date: August 21, 2020
Time: 14:00- 16:00 (Beijing Time)
Tencent Meeting ID: 970 257 901
Password: 123456

Speaker: Xu Cheng (Institute of Computational Mathematics and Scientific/Engineering Computing)
Title: Optimization methods and dynamical systems
Abstract: The optimization problem of functions has a long history, from the first order gradient algorithm O(1/ k) to second order accelerated gradient algorithm O(1/k2), from convex optimization to non-convex optimization. In recent years, more and more researchers have understood the acceleration phenomenon from the perspective of the dynamical systems.
For convex optimization, the discrete algorithm is combined with a continuous dynamical systems. (The limit case of Nesterov's acceleration gradient method is a second order ODE.) By analyzing the dissipation of this continuous dynamical systems, one can explain the phenomena of acceleration and oscillation. Later on, combined with variational methods, the convergence rate of the continuous systems can reach any polynomial degree O(1/tp); combined with the Hamiltonian systems, designing a symplectic algorithm can also obtain a better optimization algorithm.
For non-convex optimization, starting from the theory of stable manifolds, the gradient algorithm can be obtained in order to escape from a strict saddle point with probability 1. From this, a series of random algorithms (PGD, PAGD, PSGD) escaping from the saddle points are obtained by adding random perturbations. These similar discrete algorithms can be approximated by their corresponding continuous systems, that is, they can be approximated by stochastic differential equations (SDE). Extracting the dynamical systems from optimization can help to understand the deep connotation of the optimization algorithm. Furthermore, by combining the discrete and continuous systems, new algorithms can be inspired by applications of dynamical systems theory.

Date: August 28, 2020
Time: 14:00- 16:00 (Beijing Time)
Tencent Meeting ID: 923 270 635
Password: 123456

Speaker: Chongqing Cheng (Nanjing University)
Title: A toolkit for the study of dynamical instability
Abstract: In this talk, I shall introduce some techniques we invented for the construction of global connecting orbits. Some open problems shall be also proposed for further study.

Date: September 3, 2020
Time: 15:00- 16:00 (Beijing Time)
ZOOM ID: 646 976 07037
Password: 123456

Speaker: Diogo Gomes (KAUST)
Title: A price formation mean-field game model
Abstract: Here, consider a constrained mean-field game where the price is determined by a supply vs. demand balance condition. We begin by examining problems with a deterministic supply. In this case, we establish the existence of a unique solution using a fixed-point argument. In particular, we show that the price is well-defined, and it is a Lipschitz function of time. Then, we study linear-quadratic models that can be solved explicitly. Finally, we discuss the case where the supply is a random process and in the case of linear-quadratic models discuss how to solve the problem.

Date: September 4, 2020
Time: 15:00- 16:00 (Beijing Time)
ZOOM ID: 664 863 69095
Password: 123456

Speaker: Lei Zhao (University of Augsburg)
Title: KAM stability of the solar system
Abstract: We will review and comment on some mathematical theories and developments on the stability of the solar system, especially the nonlinear KAM stability.

Date: September 11, 2020
Time: 9:00- 11:00 am (Beijing Time)
Tencent Meeting ID: 803 712 343
Password: 123456

Speaker: Xianfeng Gu (State University of New York at Stony Brook)
Title: A Geometric Understanding of Deep Learning
Abstract: This work introduces an optimal transportation (OT) view of generative adversarial networks (GANs). Natural datasets have intrinsic patterns, which can be summarized as the manifold distribution principle: the distribution of a class of data is close to a low-dimensional manifold. GANs mainly accomplish two tasks: manifold learning and probability distribution transformation. The latter can be carried out using the classical OT method. From the OT perspective, the generator computes the OT map, while the discriminator computes the Wasserstein distance between the generated data distribution and the real data distribution; both can be reduced to a convex geometric optimization process. Furthermore, OT theory discovers the intrinsic collaborative—instead of competitive—relation between the generator and the discriminator, and the fundamental reason for mode collapse. We also propose a novel generative model, which uses an autoencoder (AE) for manifold learning and OT map for probability distribution transformation. This AE–OT model improves the theoretical rigor and transparency, as well as the computational stability and efficiency; in particular, it eliminates the mode collapse. The experimental results validate our hypothesis, and demonstrate the advantages of our proposed model.

Date: September 14, 2020
Time: 11:00- 12:00 (Beijing Time)
ZOOM ID: 659 865 44977
Password: 123456

Speaker: Zhiyuan Zhang (CNRS, Universite Paris 13)
Title: Exponential mixing of 3D Anosov flows
Abstract: We show that a topologically mixing C^\infty Anosov flow on a 3-dimensional compact manifold is exponential mixing with respect to any equilibrium measure with Holder potential. This is a joint work with Masato Tsujii.

Date: September 25, 2020
Time: 14:00- 15:00 (Beijing Time)
ZOOM ID: 668 041 03984
Password: 123456

Speaker: Enrico Valdinoci（University of Western Australia）
Title: Chaotic orbits for nonlocal equations, the Peierls-Nabarro model, and applications to atom dislocation dynamics in crystals
Abstract: In this talk we consider a nonlocal equation driven by a perturbed periodic potential. We construct multibump solutions that connect one integer point to another one in a prescribed way. In particular, heteroclinic, homoclinic and chaotic trajectories are constructed. This result regarding symbolic dynamics in a fractional framework is part of a study of Peierls-Nabarro model for crystal dislocations. The associated evolution equation can be studied in the mesoscopic and macroscopic limits. Namely, the dislocation function has the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior potential, which can be either repulsive or attractive, depending on the relative orientations of the dislocations. For opposite orientations, collisions occur, after which the system relaxes exponentially fast.

Date: September 28, 2020
Time: 15:00- 16:00 (Beijing Time)
ZOOM ID: 650 080 62145
Password: 123456

Speaker: Stefan Suhr（Ruhr-University Bochum）
Title: Introduction to Lorentzian Aubry-Mather Theory
Abstract: The talk will explain the geometric framework of Aubry-Mather theory in Lorentzian geometry and review the main results. If time permits possible future directions and open problems will be discussed.

Date: October 16, 2020
Time: 15:00- 16:00 (Beijing Time)
Tencent Meeting ID: 812 805 679
Password: 123456

Speaker: Guan Huang（Yau Mathematical Sciences Center）
Title: On the energy transfer to high frequencies in the damped/driven nonlinear Schrodinger equation
Abstract: We consider a damped/driven nonlinear Schr\"odinger equation in $\mathbb{R}^n$, where n is arbitrary, \[\EE u_t-\nu\Delta u+i|u|^2u=\sqrt{\nu}\eta(t,x), \quad \nu>0,\] under odd periodic boundary conditions. Here $\eta(t,x)$ is a random force which is white in time and smooth in space. It is known that the Sobolev norms of solutions satisfy $\| u(t)\|_m^2 \le C\nu^{-m},$ uniformly in $t\ge0$ and $\nu>0$. In this work we prove that for small $\nu>0$ and any initial data, with large probability the Sobolev norms $\|u(t,\cdot)\|_m$ with $m>2$ become large at least to the order of $\nu^{-\kappa_{n,m}}$ with $\kappa_{n,m}>0$, on time intervals of order $\mathcal{O}(\frac{1}{\nu})$. It proves that solutions of the equation develop short space-scale of order ν to a positive degree, and rigorously establishes the (direct) cascade of energy for the equation. This is a joint work with Sergei Kuksin.

Date: October 23, 2020
Time: 14:00- 15:00 (Beijing Time)
Tencent Meeting ID: 439 739 834
Password: 123456

Speaker: Jinxin Xue（Yau Mathematical Sciences Center）
Title: Arnold diffusion and black hole dynamics
Abstract: We consider the geodesic motions of particles in the background of a black hole, and use the methods in Hamiltonian systems to study them. We use the aperture of the black hole to get the Arnold's diffusion orbits. Arnold diffusion is a typical unstable phenomenon in nearly integrable Hamiltonian dynamics. This orbit has obvious physical meaning and can be observed. In areas far away from the horizon, we can analogize the Newtonian three-body problem to get the existence of oscillation orbits. Finally, we prove that the theory of twist mapping can be used to study the dynamics of apertures and quasi-periodic oscillations.

Date: October 30, 2020
Time: 14:00- 16:00 (Beijing Time)
ZOOM ID: 696 196 91647
Password: 123456

Speaker: Kuo-Chang Chen (National Tsing Hua University)
Title: The Kepler problem – old and new
Abstract: The Newtonian 2-body problem is also known as the Kepler problem in honor of Johannes Kepler (1571-1630) for his discovery of three laws of planetary motion, based on which Newton deduced in 1687 the celebrated law of universal gravitation. It is commonly considered a well-understood problem, as solving it with given initial data and proving Kepler's three laws require nothing more than tools from elementary calculus. In this talk I will briefly describe its history, outline recent discoveries from variational perspectives, and show some progresses regarding singularities.

Date: November 2, 2020
Time: 15:30- 16:30 (Beijing Time)
ZOOM ID: 620 998 77902
Password: 123456

Speaker: Marie-Claude Arnaud（IMJ-PRG, Université Paris-Diderot）
Title: C^0 integrability for twist map
Abstract: This is a joint work with Maxime Zavidovique. For Hamiltonian systems, the well-known Arnol’d-Liouville theorem tells us that if the system has enough C2 independent integrals, then the space is foliated by invariant Lagrangian submanifolds on which the Dynamics is conjugated to a rotation. We will consider a situation with weaker hypothesis: assume that a symplectic twist map of the annulus has an invariant foliation into continuous curve. What can be said on the this foliation and the Dynamics? After explaining some classical and less classical results in the Hamiltonian case, we will explain recent results on twist maps, e.g. that the invariant foliation is Holder, that with some other hypothesis the restricted dynamics to invariant curve is conjugate to a rotation.

Date: November 16, 2020
Time: 16:00- 17:00 (Beijing Time)
ZOOM ID: 622 098 20563
Password: 123456

Speaker: Alex Haro（University of Barcelona）
Title: Singularity Theory for non-twist tori
Abstract: We present a method to find nontwist KAM tori. These are tori for which the twist condition fails. Our method also leads to a natural classification of KAM tori which is based on Singularity Theory. This talk aims to illustrate the main ideas of our approach, going from rigorous results to numerical computations up to the verge of breakdown. This a joint project with Rafael de la Llave and Alejandra González.

Date: November 20, 2020
Time: 9:00- 11:00 (Beijing Time)
ZOOM ID: 687 466 26905
Password: 123456

Speaker 1: Renato Calleja (National Autonomous University of Mexico)
Title: Whiskered KAM Tori of Conformally Symplectic Systems
Abstract: Many physical problems are described by conformally symplectic systems. We study the existence of whiskered tori in a family f_μ of conformally symplectic maps depending on parameters μ. Whiskered tori are tori on which the motion is a rotation but having as many contracting/expanding directions as allowed by the preservation of the geometric structure.

Our main result is formulated in an a-posteriori format. Given an approximately invariant embedding of the torus for a parameter value μ_0 with an approximately invariant splitting, there is an invariant embedding and invariant splittings for new parameters.

Using the results of formal expansions as the starting point for the a-posteriori method, we study the domains of analiticity of parameterizations of whiskered tori in perturbations of Hamiltonian Systems with dissipation. The proofs of the results lead to efficient algorithms that are quite practical to implement.

Joint work with A. Celletti and R. de la Llave, A.P. Bustamante.

Speaker 2: Rodrigo Treviño (University of Maryland)
Title: Quasicrystals, aperiodic tilings, and their interaction with dynamical systems
Abstract: The world of aperiodic tilings is a meeting point of many disciplines such as mathematical physics, dynamical systems, operator algebras, discrete geometry, to name a few. In this talk I will give go over the origins of the topic, some important areas of research, recent results and open questions. I will emphasize the important role that tools from dynamical systems and ergodic theory play in all of it.

Date: November 30, 2020
Time: 15:00- 16:00 (Beijing Time)
ZOOM ID: 666 349 41827
Password: 123456

Speaker: Florian Kogelbauer（ETH Zurich）
Title: On the Relation between Conservative and Nonconservative Nonlinear Normal Modes
Abstract: The Lyapunov subcenter manifold, a two-dimensional invariant manifold filled with periodic orbits, is a classical object in the study of normal modes in conservative systems. Recently, the existence of invariant manifolds in dissipative systems has been successfully applied to model reduction and the computation of backbone curves. In this talk, bridging these two mutually exclusive settings, we will present a theorem on the persistence of the Lyapunov subcenter manifold under autonomous perturbations. We will show applications to an infinite-dimensional beam equation and the computation of instantaneous backbone curves.

Date: December 21, 2020
Time: 22:00- 23:00 (Beijing Time)
ZOOM ID: 680 391 94059
Password: 123456

Speaker: Renato Iturriaga（CIMAT）
Title: Discrete Approximation of stochastic Mather Measures and the viscous Hamilton Jacobi equation
Abstract: We consider a stochastic discretization of the stationary viscous Hamilton- Jacobi equation on the flat d–dimensional torus T^d associated with a Hamiltonian, convex and superlinear in the momentum variable. We show that each discrete problem admits a unique continuous solution on T^d, up to additive constants and a unique stochastic measure. We show that as the step goes to zero converges to the stochastic Mather measure. By additionally assuming a technical condition on the associated Lagrangian, we show that each solution of the viscous Hamilton–Jacobi equation is the limit of solutions of the discrete problems.

Date: December 22, 2020
Time: 09:30- 10:30 (Beijing Time)
ZOOM ID: 636 918 43731
Password: 123456

Speaker: Rafael de la Llave（Georgia Institute of Technology）
Title: Geometric objects in Hamiltonian systems with weak dissipation and weak forcing
Abstract: Many practical systems (e.g. in celestial mechanics, mechanical engineering) can be described as hamiltonian systems modified by small friction. Even adding a small friction changes the very long term behavior very drastically, many features of the motion can be analyzed.

I will describe two recent works:
1. If the Hamiltonian system has a KAM torus, it is possible to obtain asymptotic expansions in the friction of the needed forcing so that the perturbed system has a quasi-periodic solution of the same frequency. These series do not converge, but indeed, there is a quasi periodic solution with a complicated domain of analyticity. We study these series and show that they are Gevery. (Joint work with A. Perez Bustamante)
2. If the Hamiltonian systems has separatrices (orbits for which the stable and unstable manifolds agree). A general perturbation may lead to transverse homoclinic intersections. We describe generalizations of the classical Melnikov method to dissipative perturbations. (Joint work with M. Gidea and M. Musser)