The Second Homework

March 24, 2025

1. Compute the truth tables for

   (a) $p \to (\neg p \wedge q) \vee q$.

   (b) $(\neg p \to q) \wedge q \wedge r$.

   SOLUTION:

   (a) $p \to (\neg p \wedge q) \vee q$.

p	q	$\neg p$	$\neg p \wedge q$	$(\neg p \wedge q) \vee q$	$p \to (\neg p \wedge q) \vee q$
1	1	0	0	1	1
1	0	0	0	0	0
0	1	1	1	1	1
0	0	1	0	0	1

   (b) $(\neg p \to q) \wedge q \wedge r$.

p	q	r	$\neg p$	$\neg p \to q$	$(\neg p \to q) \wedge q \wedge r$
1	1	1	0	1	1
1	1	0	0	1	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	1	1
0	1	0	1	1	0
0	0	1	1	0	0
0	0	0	1	0	0

2. Show that the following are tautologies:

   (a) $\neg\neg p \to p$.

   (b) $(p \to q) \to (\neg q \to \neg p)$.

   (c) $(p \to q) \leftrightarrow (\neg p \vee q)$.

<u>SOLUTION</u>:

(a) $\neg\neg p \to p$.

p	$\neg p$	$\neg\neg p$	$\neg\neg p \to p$
1	0	1	1
0	1	0	1

(b) $(p \to q) \to (\neg q \to \neg p)$.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$	$(p \to q) \to (\neg q \to \neg p)$
1	1	0	0	1	1	1
1	0	0	1	0	0	1
0	1	1	0	1	1	1
0	0	1	1	1	1	1

(c) $(p \to q) \leftrightarrow (\neg p \vee q)$.

p	q	$\neg p$	$p \to q$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
1	1	0	1	1	1
1	0	0	0	0	1
0	1	1	1	1	1
0	0	1	1	1	1

3. Show that $(p \to r) \vee (q \to r) \Leftrightarrow (p \wedge q) \to r$.

<u>SOLUTION</u>:

$$\begin{aligned} (p \to r) \vee (q \to r) &\Leftrightarrow (\neg p \vee r) \vee (\neg q \vee r) \\ &\Leftrightarrow (\neg p \vee \neg q) \vee r \\ &\Leftrightarrow \neg(p \wedge q) \vee r \\ &\Leftrightarrow (p \wedge q) \to r. \end{aligned}$$

4. Find equivalent normal disjunction formulas for:

   (a) $(p \wedge q) \vee r$.

   (b) $(\neg p \to q) \wedge q \wedge r$.

SOLUTION: First get the basic conjunction terms via the truth table.

m_i	basic conjunction terms	p	q	r	$(p \wedge q) \vee r$	$(\neg p \to q) \wedge q \wedge r$
m_{000}	$\neg p \wedge \neg q \wedge \neg r$	0	0	0	0	0
m_{001}	$\neg p \wedge \neg q \wedge r$	0	0	1	1	0
m_{010}	$\neg p \wedge q \wedge \neg r$	0	1	0	0	0
m_{011}	$\neg p \wedge q \wedge r$	0	1	1	1	1
m_{100}	$p \wedge \neg q \wedge \neg r$	1	0	0	0	0
m_{101}	$p \wedge \neg q \wedge r$	1	0	1	1	0
m_{110}	$p \wedge q \wedge \neg r$	1	1	0	1	0
m_{111}	$p \wedge q \wedge r$	1	1	1	1	1

Then take disjunction of the terms corresponding to "1" to get the equivalent normal form:

(a) $(p \wedge q) \vee r \Leftrightarrow (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r)$.

(b) $(\neg p \to q) \wedge q \wedge r \Leftrightarrow (\neg p \wedge q \wedge r) \vee (p \wedge q \wedge r)$. ■

5. Show that $\{\uparrow\}$ and $\{\downarrow\}$ are complete.

SOLUTION:

(a) Since $p \uparrow p \Leftrightarrow \neg(p \wedge p) \Leftrightarrow \neg p$ and $p \uparrow \neg q \Leftrightarrow \neg(p \wedge \neg q) \Leftrightarrow \neg p \vee q \Leftrightarrow p \to q$, we can know that $\{\uparrow\}$ is complete because $\{\neg, \to\}$ is complete.

(b) Since $p \downarrow p \Leftrightarrow \neg(p \vee p) \Leftrightarrow \neg p$ and $\neg(\neg p \downarrow q) \Leftrightarrow \neg p \vee q \Leftrightarrow p \to q$, we can know that $\{\downarrow\}$ is complete because $\{\neg, \to\}$ is complete. ■