

Poisson representations of measure-valued processes

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with Eliane Rodrigues [5]



Poisson random measures

S a Polish space and ν a σ -finite measure on $\mathcal{B}(S)$.

ξ is a *Poisson random measure* with mean measure ν if

- a) ξ is a random counting measure on S .
- b) For each $A \in \mathcal{S}$ with $\nu(A) < \infty$, $\xi(A)$ is Poisson distributed with parameter $\nu(A)$.
- c) For $A_1, A_2, \dots \in \mathcal{S}$ disjoint, $\xi(A_1), \xi(A_2), \dots$ are independent.

Lemma 1 *If $\nu(A) < \infty$, then conditioned on $\xi(A)$, the point process $\xi(\cdot \cap A)$ has the same distribution as $\sum_{i=1}^{\xi(A)} \delta_{\zeta_i}$, where $\zeta_1, \dots, \zeta_{\xi(A)}$ are iid with distribution $\nu(A)^{-1}\nu(\cdot \cap A)$.*

If $H : S \rightarrow S_0$, Borel measurable, and $\hat{\xi}(A) = \xi(H^{-1}(A))$, then $\hat{\xi}$ is a Poisson random measure on S_0 with mean measure $\hat{\nu}$ given by $\hat{\nu}(A) = \nu(H^{-1}(A))$



Moment identities

If ξ is a Poisson random measure with mean measure ν

$$E[e^{\int f(z)\xi(dz)}] = e^{\int (e^f - 1)d\nu},$$

or letting $\xi = \sum_i \delta_{Z_i}$,

$$E\left[\prod_i g(Z_i)\right] = e^{\int (g-1)d\nu}.$$

Similarly,

$$E\left[\sum_j h(Z_j) \prod_i g(Z_i)\right] = \int hgd\nu e^{\int (g-1)d\nu},$$

and

$$E\left[\sum_{i \neq j} h(Z_i)h(Z_j) \prod_k g(Z_k)\right] = \left(\int hgd\nu\right)^2 e^{\int (g-1)d\nu},$$



Conditionally Poisson systems

Let ξ be a random counting measure on S and Ξ be a locally finite random measure on S .

ξ is *conditionally Poisson* with Cox measure Ξ if, conditioned on Ξ , ξ is a Poisson point process with mean measure Ξ .

$$E[e^{-\int_S f d\xi}] = E[e^{-\int_S (1-e^{-f}) d\Xi}]$$

for all nonnegative $f \in M(S)$.

If ξ is conditionally Poisson system on $S \times [0, \infty)$ with Cox measure $\Xi \times m$ where m is Lebesgue measure, then for $f \in M(S)$

$$E[e^{-\int_{S \times [0, K]} f d\xi}] = E[e^{-K \int_S (1-e^{-f}) d\Xi}]$$

and for $f \geq 0$,

$$\Xi(f) = \lim_{K \rightarrow \infty} \frac{1}{K} \int_{S \times [0, K]} f d\xi = \lim_{\epsilon \rightarrow 0} \epsilon \int_{S \times [0, \infty)} e^{-\epsilon u} f(x) \xi(dx \times du) \quad a.s.$$



Relationship to exchangeability

Lemma 2 *Suppose ξ is a conditionally Poisson random measure on $S \times [0, \infty)$ with Cox measure $\Xi \times m$. If $\Xi < \infty$ a.s., then we can write $\xi = \sum_{i=1}^{\infty} \delta_{(X_i, U_i)}$ with $U_1 < U_2 < \dots$ a.s. and $\{X_i\}$ is exchangeable.*

Conditioned on Ξ , $\{U_i\}$ is Poisson with parameter $\Xi(S)$ and $\{X_i\}$ is iid with distribution $\Xi(S)^{-1}\Xi$.



Convergence

Lemma 3 *If $\{\xi_n\}$ is a sequence of conditionally Poisson random measures on $S \times [0, \infty)$ with Cox measures $\{\Xi_n \times m\}$. Then $\xi_n \Rightarrow \xi$ if and only if $\Xi_n \Rightarrow \Xi$, Ξ the Cox measure for ξ .*

If $\xi_n \rightarrow \xi$ in probability, then $\Xi_n \rightarrow \Xi$ in probability



A population model

Consider a process with state space $E = \cup_n [0, r]^n$.

$0 \leq g \leq 1$, $g(r) = 1$, and $f(u, n) = \prod_{i=1}^n g(u_i)$

For $a > 0$, and $-\infty < b \leq ra$, define

$$A_r f(u, n) = f(u, n) \sum_{i=1}^n 2a \int_{u_i}^r (g(v) - 1) dv + f(u, n) \sum_{i=1}^n (au_i^2 - bu_i) \frac{g'(u_i)}{g(u_i)}.$$

In other words, particle levels satisfy

$$\dot{U}_i(t) = aU_i^2(t) - bU_i(t),$$

and a particle with level z gives birth at rate $2a(r - z)$ to a particle whose initial level is uniformly distributed between z and r .

$$N(t) = \#\{i : U_i(t) < r\}$$

$\alpha_r(n, du)$ the joint distribution of n iid uniform $[0, r]$ random variables.



A calculation

$$\widehat{f}(n) = \int f(u, n) \alpha_r(n, du) = e^{-\lambda_g n}, \quad e^{-\lambda_g} = \frac{1}{r} \int_0^r g(u) du$$

To calculate $C\widehat{f}(n) = \int A_r f(u, n) \alpha_r(n, du)$, observe that

$$r^{-1} 2a \int_0^r g(z) \int_z^r (g(v) - 1) dv = ar e^{-2\lambda_g} - 2ar^{-1} \int_0^r g(z)(r - z) dz$$

and

$$\begin{aligned} r^{-1} \int_0^r (az^2 - bz) g'(z) dz &= -r^{-1} \int_0^r (2az - b)(g(z) - 1) dz \\ &= -2ar^{-1} \int_0^r zg(z) dz + ar + b(e^{-\lambda_g} - 1). \end{aligned}$$

Then

$$\begin{aligned} C\widehat{f}(n) &= ne^{-\lambda_g(n-1)} (are^{-2\lambda_g} - 2are^{-\lambda_g} + ar + b(e^{-\lambda_g} - 1)) \\ &= arn(\widehat{f}(n+1) - \widehat{f}(n)) + (ar - b)n(\widehat{f}(n-1) - \widehat{f}(n)). \end{aligned}$$



Conclusion

Let \tilde{N} be a solution of the martingale problem for

$$C\hat{f}(n) = arn(\hat{f}(n+1) - \hat{f}(n)) + (ar - b)n(\hat{f}(n-1) - \hat{f}(n)),$$

that is, \tilde{N} is a branching process with birth rate ar and death rate $(ar - b)$.

Then there exists a solution $(U_1(t), \dots, U_{N(t)}(t), N(t))$ of the martingale problem for A such that N has the same distribution as \tilde{N} .



The limit as $r \rightarrow \infty$

If $n = O(r)$, then the scaling is correct for the Feller diffusion.

For $f(u) = \prod_i g(u_i)$, $0 \leq g \leq 1$, $g(z) = 1$, $z \geq u_g$, $A_r f$ converges to

$$Af(u) = f(u) \sum_i 2a \int_{u_i}^{u_g} (g(v) - 1)dv + f(u) \sum_i (au_i^2 - bu_i) \frac{g'(u_i)}{g(u_i)}.$$

If $nr^{-1} \rightarrow y$, then $\alpha_r(n, du) \rightarrow \alpha(y, du)$ where $\alpha(y, du)$ is the distribution of a Poisson process on $[0, \infty)$ with intensity y .

$$\widehat{f}(y) = \alpha f(y) = \int f(u) \alpha(y, du) = e^{-y \int_0^\infty (1-g(z))dz} = e^{-y\beta_g}$$

and

$$\begin{aligned} \alpha Af(y) &= e^{-y\beta_g} \left(2ay \int_0^\infty g(z) \int_z^\infty (g(v) - 1)dv dz + y \int_0^\infty (az^2 - bz)g'(z)dz \right) \\ &= e^{-y\beta_g} (ay\beta_g^2 - by\beta_g) \\ &= ay\widehat{f}''(y) + by\widehat{f}'(y) \end{aligned}$$



Particle representation of Feller diffusion

Let $\{U_i(0)\}$ be a conditionally Poisson process on $[0, \infty)$ with (conditional) intensity $Y(0)$. Then, $\{U_i(t)\}$ is conditionally Poisson with intensity $Y(t)$,

$$Y(t) = \lim_{r \rightarrow \infty} \frac{1}{r} \#\{i : U_i(t) \leq r\},$$

and Y is a Feller diffusion with generator $Cf(y) = ayf''(y) + byf'(y)$

$$\gamma : \mathcal{N}(\mathbb{R}) \rightarrow [0, \infty)$$

$$\gamma(u) = \lim_{r \rightarrow \infty} \frac{1}{r} \#\{i : u_i \leq r\}$$

$\alpha(y, du)$ Poisson process distribution on $[0, \infty)$ with intensity y .

$$\alpha(y, \gamma^{-1}(y)) = 1.$$



Extinction

Assume $U_1(0) < U_2(0) < \dots$. Then for all t , all levels are above

$$U_1(t) = \frac{U_1(0)e^{-bt}}{1 - \frac{a}{b}U_1(0)(1 - e^{-bt})}$$

Let $\tau = \inf\{t : Y(t) = 0\}$

$$P\{\tau > t\} = P\{U_1(0) < [(1 - e^{-bt})a/b]^{-1}\} = 1 - e^{-yb/[(1 - e^{-bt})a]}$$

If $b \leq 0$, conditioning on nonextinction for all t is equivalent to setting $U_1(0) = 0$. The generator becomes

$$\begin{aligned} Af(u) = f(u) \sum_i 2a \int_{u_i}^{u_g} (g(v) - 1)dv + f(u) \sum_i (au_i^2 - bu_i) \frac{g'(u_i)}{g(u_i)} \\ + f(u) 2a \int_0^{u_g} (g(v) - 1)dv \end{aligned}$$

and

$$\alpha Af(y) = ay\hat{f}''(y) + (2a + by)\hat{f}'(y)$$



Branching Markov processes

$f(x, u, n) = \prod_{i=1}^n g(x_i, u_i)$, where $g : E \times [0, \infty) \rightarrow (0, 1]$

As a function of x , g is in the domain $\mathcal{D}(B)$ of the generator of a Markov process in E , g is continuously differentiable in u , and $g(x, u) = 1$ for $u \geq r$.

$$\begin{aligned} Af(x, u, n) &= f(x, u, n) \sum_{i=1}^n \frac{Bg(x_i, u_i)}{g(x_i, u_i)} \\ &\quad + f(x, u, n) \sum_{i=1}^n 2a(x_i) \int_{u_i}^r (g(x_i, v) - 1) dv \\ &\quad + f(x, u, n) \sum_{i=1}^n (a(x_i)u_i^2 - b(x_i)u_i) \frac{\partial_{u_i} g(x_i, u_i)}{g(x_i, u_i)} \end{aligned}$$

Each particle has a location $X_i(t)$ in E and a level $U_i(t)$ in $[0, r]$.



Behavior of the process

The locations evolve independently as Markov processes with generator B , the levels satisfy

$$\dot{U}_i(t) = a(X_i(t))U_i^2(t) - b(X_i(t))U_i(t)$$

and particles that reach level r die.

Particles give birth at rates $2a(X_i(t))(r - U_i(t))$; the initial location of a new particle is the location of the parent at the time of birth; and the initial level is uniformly distributed on $[U_i(t), r]$.



Generator for $X(t) = (X_1(t), \dots, X_N(t))$

Setting $e^{-\lambda_g(x_i)} = r^{-1} \int_0^r g(x_i, z) dz$ and $\hat{f}(x, n) = e^{-\sum_{i=1}^n \lambda_g(x_i)}$, and calculating as in the previous example, we have

$$\begin{aligned} C\hat{f}(x, n) &= \sum_{i=1}^n B_{x_i} \hat{f}(x, n) + \sum_{i=1}^n a(x_i) r (\hat{f}((x, x_i), n+1) - \hat{f}(x, n)) \\ &\quad + \sum_{i=1}^n (a(x_i) r - b(x_i)) (\hat{f}(d(x|x_i), n-1) - \hat{f}(x, n)), \end{aligned}$$

where B_{x_i} is the generator B applied to $\hat{f}(x, n)$ as a function of x_i and $d(x|x_i)$ is the vector obtained from x by eliminating the i th component.



Infinite population limit

Letting $r \rightarrow \infty$, Af becomes

$$Af(x, u) = f(x, u) \sum_i \frac{Bg(x_i, u_i)}{g(x_i, u_i)} + f(x, u) \sum_i 2a(x_i) \int_{u_i}^{u_g} (g(x_i, v) - 1)dv \\ + f(x, u) \sum_i (a(x_i)u_i^2 - b(x_i)u_i) \frac{\partial_{u_i} g(x_i, u_i)}{g(x_i, u_i)}$$

Particle locations evolve as independent Markov processes with generator B , and levels satisfy

$$\dot{U}_i(t) = a(X_i(t))U_i^2(t) - b(X_i(t))U_i(t)$$

A particle with level $U_i(t)$ gives birth to new particles at its location $X_i(t)$ and initial level in the interval $[U_i(t) + c, U_i(t) + d]$ at rate $2a(X_i(t))(d - c)$.

A particle dies when its level hits ∞ .



The measure-valued limit

For $\mu \in \mathcal{M}_f(E)$, let $\alpha(\mu, dx \times du)$ be the distribution of a Poisson random measure on $E \times [0, \infty)$ with mean measure $\mu \times m$. Then setting $h(y) = \int_0^\infty (1 - g(y, v)) dv$

$$\alpha f(\mu) = \int f(x, u) \alpha(\mu, dx \times du) = \exp\left\{-\int_E h(y) \mu(dy)\right\},$$

and

$$\begin{aligned} \alpha Af(\mu) &= \exp\left\{-\int_E h(y) \mu(dy)\right\} \left[\int_E \int_0^\infty Bg(y, v) dv \mu(dy) \right. \\ &\quad + \int_E \int_0^\infty 2a(y)g(y, z) \int_z^\infty (g(y, v) - 1) dv dz \mu(dy) \\ &\quad \left. + \int_E \int_0^\infty (a(y)v^2 - b(y)v) \partial_v g(y, v) dv \mu(dy) \right] \\ &= \exp\left\{-\int_E h(y) \mu(dy)\right\} \int_E (-Bh(y) + a(y)h(y)^2 - b(y)h(y)) \mu(dy) \end{aligned}$$



Representation of the Dawson-Watanabe process

It follows that the Cox measure (or more precisely, the E marginal of the Cox measure) corresponding to the particle process at time t , call it $Z(t)$, is a solution of the martingale problem for

$$\mathbb{A} = \{(\alpha f, \alpha A f) : f \in \mathcal{D}\}.$$



Branching processes in random environments[2, 3, 4]

a and b functions of another stochastic process η , say an irreducible finite Markov chain with generator Q .

$$f(l, u, n) = f_0(l)f_1(u) = f_0(l) \prod_{i=1}^n g(u_i),$$

$$\begin{aligned} A_r f(l, u, n) &= r f_1(u) Q f_0(l) + f(l, u, n) \sum_{i=1}^n 2a(l) \int_{u_i}^r (g(v) - 1) dv \\ &\quad + f(l, u, n) \sum_{i=1}^n (a(l)u_i^2 - \sqrt{r}b(l)u_i) \frac{g'(u_i)}{g(u_i)}, \end{aligned}$$

which projects to

$$\begin{aligned} C_r \hat{f}(l, n) &= r Q \hat{f}(l, n) + a(l) r n (\hat{f}(l, n+1) - \hat{f}(l, n)) \\ &\quad + (r a(l) - \sqrt{r} b(l)) n (\hat{f}(l, n-1) - \hat{f}(l, n)), \end{aligned}$$

where $\hat{f}(l, n) = f_0(l) e^{-\lambda_g n}$. The process corresponding to C_r is a branching process in a random environment determined by η .



Scaling limit

Writing the process corresponding to A_r as

$$(\eta(rt), U_1(t), \dots, U_{N_r(t)})$$

the process corresponding to C_r is $(\eta(rt), N_r(t))$.

Note that the levels satisfy

$$\dot{U}_i(t) = a(\eta(rt))U_i^2(t) - \sqrt{r}b(\eta(rt))U_i(t).$$

Let π be the stationary distribution for Q . Assume that $\sum_l \pi(l)b(l) = 0$ and $Qh_0(l) = b(l)$. Then

$$W^{(r)}(t) = \sqrt{r} \int_0^t b(\eta(rs)) ds$$

converges to a Brownian motion W with variance parameter

$$\sum_k \sum_l \pi(k) q_{kl} (h_0(l) - h_0(k))^2 = -2 \sum_l \pi(l) h_0(l) b(l) \equiv 2\bar{c}.$$



Limiting level processes

In the limit, the levels will satisfy

$$dU_i(t) = (\bar{a}U_i(t)^2 + \bar{c}U_i(t))dt + \sqrt{2\bar{c}}U_i(t)dW(t), \quad (1)$$

where $\bar{a} = \sum \pi(l)a(l)$.

Note that the limiting levels are all driven by the same Brownian motion.



Limiting generator

$$\begin{aligned} Af_1(u) &= f_1(u) \sum_i 2\bar{a} \int_{u_i}^{\infty} (g(v) - 1) dv \\ &+ f_1(u) \sum_i \frac{(\bar{c}u_i + \bar{a}u_i^2)g'(u_i) + \bar{c}u_i^2g''(u_i)}{g(u_i)} \\ &+ \bar{c}f_1(u) \sum_{j=1}^n \left(\sum_{i \neq j} u_j u_i \frac{g'(u_i)g'(u_j)}{g(u_i)g(u_j)} \right). \end{aligned}$$



Limiting diffusion

Let

$$\beta_g = \int_0^\infty (1 - g(z))dz = \int_0^\infty zg'(z)dz = -\frac{1}{2} \int_0^\infty z^2 g''(z)dz.$$

We have

$$\begin{aligned} \alpha Af(y) &= e^{-y\beta_g} (2\bar{a}y \int_0^\infty g(z) \int_z^\infty (g(v) - 1)dv dz + y \int_0^\infty (\bar{a}z^2 + \bar{c}z)g'(z)dz \\ &\quad + \bar{c}y^2 (\int_0^\infty zg'(z)dz)^2 + \bar{c}y \int_0^\infty z^2 g''(z)dz) \\ &= e^{-y\beta_g} ((\bar{a}y + \bar{c}y^2)\beta_g^2 - \bar{c}y\beta_g) \\ &= C\hat{f}(y), \end{aligned}$$

where

$$C\hat{f}(y) = (\bar{a}y + \bar{c}y^2)\hat{f}''(y) + \bar{c}y\hat{f}'(y),$$

which identifies the diffusion limit for $r^{-1}N_r$.



Exit measures [1, 6]

Let B be the generator of a diffusion and D a bounded domain with all points in ∂D regular.

$$\begin{aligned} Af(x, u) &= f(x, u) \sum_i 1_D(x_i) \frac{Bg(x_i, u_i)}{g(x_i, u_i)} \\ &\quad + f(x, u) \sum_i 1_D(x_i) 2a(x_i) \int_{u_i}^{u_g} (g(x_i, v) - 1) dv \\ &\quad + f(x, u) \sum_i 1_D(x_i) (a(x_i)u_i^2 - b(x_i)u_i) \frac{\partial_{u_i} g(x_i, u_i)}{g(x_i, u_i)} \end{aligned}$$

Assume $\inf_x a(x) > 0$, $b \leq 0$, then $\tau = \inf\{t : \sum_i 1_D(X_i(t)) = 0\} < \infty$. Define $\xi = \sum_i \delta_{(X_i(\tau), U_i(\tau))}$, and define the *exit measure*

$$Z_D(\Gamma) = \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{i: U_i(\tau) \leq r} \mathbf{1}_\Gamma(X_i(\tau)),$$



Nonlinear PDE

Assume $\{(X_i(0), U_i(0))\}$ is a Poisson random measure with mean measure $\delta_x \times m$. Then for $h \geq 0$, bounded and continuous,

$$u(x) = -\log E[e^{-\langle h, Z_D^x \rangle}]$$

solves the differential equation

$$Bu = au^2 - bu$$

$$u = h \text{ on } \partial D.$$

$$e^{-\langle h, Z_D^x \rangle} = E[e^{\int \log(1-h/r) 1_{[0,r]} d\xi} | Z_D] = E[e^{\sum_{U_i(\tau) \leq r} \log(1-h(X_i(\tau))/r)} | Z_D].$$

Taking expectations,

$$E[e^{-\langle h, Z_D^x \rangle}] = E\left[\prod_{U_i(\tau) \leq r} \left(1 - \frac{h(X_i(\tau))}{r}\right)\right]$$



Homogenization

$$\begin{aligned} Bu(x) &= a(x, x/\epsilon)u(x)^2 - b(x, x/\epsilon)u(x) \\ u &= h \text{ on } \partial D \end{aligned}$$

$a(x, y)$, $b(x, y)$ periodic in y , and

$$Bg(x) = \frac{1}{2} \sum_{j,k=1}^d c_{jk}(x) \frac{\partial^2 g}{\partial x_j \partial x_k}(x) + \sum_{j=1}^d d_j(x) \frac{\partial g}{\partial x_j}(x).$$

Particle generator:

$$\begin{aligned} A^\epsilon f(x, u) &= f(x, u) \sum_i \frac{Bg(x_i, u_i)}{g(x_i, u_i)} 1_D(x_i) \\ &+ f(x, u) \sum_i 2a(x_i, x_i/\epsilon) \int_{u_i}^{u_g} (g(x_i, v) - 1) dv 1_D(x_i) \\ &+ f(x, u) \sum_i (a(x_i, x_i/\epsilon)u_i^2 - b(x_i, x_i/\epsilon)u_i) \frac{\partial_{u_i} g(x_i, u_i)}{g(x_i, u_i)} 1_D(x_i) \end{aligned}$$



Effective coefficients

Let $\bar{a}(x) = \int a(x, y)\pi_x(dy)$, $\bar{b}(x) = \int b(x, y)\pi_x(dy)$, where π_x is determined by

$$\int \widehat{B}g(x, y)\pi_x(dy) = 0, \quad g \in \mathbb{R}^d,$$

where

$$\widehat{B}g(x, y) = \frac{1}{2} \sum_{j,k=1}^d c_{jk}(x) \frac{\partial^2 g}{\partial y_j \partial y_k}(y).$$

$$\lim_{\epsilon \rightarrow 0} E[e^{-\langle Z_D^{\epsilon, x}, h \rangle}] = \lim_{\epsilon \rightarrow 0} E\left[\prod_{U_i^\epsilon(\tau) \leq r} \left(1 - \frac{h(X_i(\tau))}{r}\right) \right] = E[e^{-\langle \bar{Z}_D^x, h \rangle}]$$



Conditioning on non-extinction

Let a be constant and $b \equiv 0$. and let $U_*(0)$ be the minimum of the initial levels.

$$U_*(t) = \frac{U_*(0)}{1 - aU_*(0)t}.$$

Let τ be the time of extinction, Then $\{\tau > T\} = \{U_*(0) < \frac{1}{aT}\}$. Conditioning on $\{\tau > T\}$ and letting $T \rightarrow \infty$ is equivalent to conditioning on the initial Poisson process having a level at zero. The resulting generator becomes

$$\begin{aligned} Af(x, u) = f(x, u) \sum_i \frac{Bg(x_i, u_i)}{g(x_i, u_i)} + f(x, u) \sum_{i>0} 2a \int_{u_i}^{r_g} (g(x_i, v) - 1)dv \quad (2) \\ + f(x, u) 2a \int_0^{r_g} (g(x_0, v) - 1)dv \\ + f(x, u) \sum_{i>0} au_i^2 \frac{\partial_{u_i} g(x_i, u_i)}{g(x_i, u_i)}, \end{aligned}$$



Generator for measure-valued process

The generator for the measure-valued process is given by setting

$$\alpha_0 f(\mu) = \int f(x, u) \alpha_0(\mu, dx \times du) = \frac{1}{|\mu|} \int_E g(z, 0) \mu(dz) \exp\{-\langle h, \mu \rangle\},$$

and

$$\begin{aligned} \alpha_0 A f(\mu) &= \langle -Bh(y) + ah(y)^2, \mu \rangle \frac{1}{|\mu|} \int_E g(z, 0) \mu(dz) \exp\{-\langle h, \mu \rangle\} \\ &\quad + \frac{1}{|\mu|} \int_E (Bg(z, 0) - 2ag(z, 0)h(z)) \mu(dz) \exp\{-\langle h, \mu \rangle\} \end{aligned}$$



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Abstract

Poisson representations of measure-valued processes

Measure-valued diffusions and measure-valued solutions of stochastic partial differential equations can be represented in terms of the Cox measures of particle systems that are conditionally Poisson at each time t . The representations are useful for characterizing the processes, establishing limit theorems, and analyzing the behavior of the measure-valued processes. Examples will be given and some of the useful methodology will be described.

