(Updated: 2012/05/06)

A remark on Chapter 10 of

"Measure-Valued Branching Markov Processes" by Li (Springer, 2011)

For most of the results in Chapter 10, the second moment assumption on the branching mechanism is unnecessary. Only the following modifications are needed:

• (Page 233, lines –3 and –4 in the first paragraph) Delete "that $\nu(1)^2 H(x, \mathrm{d}\nu)$ is a bounded kernel from E to $M(E)^\circ$ and".

• (Page 236, line –4) At the beginning of Corollary 10.6 add "Suppose that $\nu(1)^2 H(x, \mathrm{d}\nu)$ is a bounded kernel from E to $M(E)^\circ$."

• (Page 237, line 2) At the beginning of Corollary 10.7 replace "There is" by "Suppose that $\nu(1)^2 H(x, \mathrm{d}\nu)$ is a bounded kernel from E to $M(E)^\circ$. Let $\{Y_t : t \geq 0\}$ be defined by (10.7). Then there is"

• (Page 238, starting from line –5) Replace "By applying...where $\|\kappa\| = \|\kappa(\cdot, 1)\|$." by "By Lemma 10.9, the result of Theorem 10.4 can be applied to the process $\{Z_{j,k}(t) : t \geq 0\}$. Then following the arguments in Sections 7.2 and 9.3 one can see $\{Z_{j,k}(t) : t \geq 0\}$ has no negative jumps. Furthermore, it has the following properties:

(a) Let $N(\mathrm{d}s, \mathrm{d}\nu)$ be the optional random measure on $[0, \infty) \times M(E)^\circ$ defined by

$$N(\mathrm{d}s, \mathrm{d}\nu) = \sum_{s>0} 1_{\{\Delta Z_{j,k}(s) \neq 0\}} \delta_{(s, \Delta Z_{j,k}(s))}(\mathrm{d}s, \mathrm{d}\nu),$$

where $\Delta Z_{j,k}(s) = Z_{j,k}(s) - Z_{j,k}(s-)$, and let $\hat{N}(\mathrm{d}s, \mathrm{d}\nu)$ denote the predictable compensator of $N(\mathrm{d}s, \mathrm{d}\nu)$. Then $\hat{N}(\mathrm{d}s, \mathrm{d}\nu) = \mathrm{d}s K(Z_{j,k}(s-), \mathrm{d}\nu)$ with

$$K(\mu, \mathrm{d}\nu) = \int_E \mu(\mathrm{d}x) H(x, \mathrm{d}\nu).$$

(b) Let $\tilde{N}(\mathrm{d}s, \mathrm{d}\nu) = N(\mathrm{d}s, \mathrm{d}\nu) - \hat{N}(\mathrm{d}s, \mathrm{d}\nu)$ and $\rho_s(y) = |\rho_k(s, y) - \rho_j(s, y)|$. Then for any $f \in D_0(A)$,

$$\langle Z_{j,k}(t), f \rangle = M_t^c(f) + M_t^d(f) + \int_0^t [\langle Z_{j,k}(s), Af + \gamma f - bf \rangle + \langle \lambda, \rho_s \kappa f \rangle] \mathrm{d}s,$$

where $\{M_t^c(f) : t \geq 0\}$ is a continuous local martingale with quadratic variation $2\langle Z_{j,k}(t), cf^2\rangle \mathrm{d}t$ and

$$M_t^d(f) = \int_0^t \int_{M(E)^\circ} \langle \nu, f\rangle \tilde{N}(\mathrm{d}s, \mathrm{d}\nu), \quad t \geq 0,$$

is a purely discontinuous local martingale.

As in the proof of Theorem 7.14, using Hölder's inequality and Doob's martingale inequality we get

$$\begin{aligned}
\mathbf{P}\Big[\sup_{0\leq s\leq t}\langle Z_{j,k}(s),1\rangle\Big] \leq\ & \mathbf{P}\Big[\sup_{0\leq s\leq t}|M_s^c(1)|\Big] + \|\kappa\|\mathbf{P}\Big[\int_0^t \langle\lambda,\rho_s\rangle \mathrm{d}s\Big] \\
& + \mathbf{P}\Big[\sup_{0\leq s\leq t}\Big|\int_0^s \int_{M(E)^\circ} \langle\nu,1\rangle 1_{\{\langle\nu,1\rangle\leq 1\}}\tilde{N}(\mathrm{d}r,\mathrm{d}\nu)\Big|\Big] \\
& + \mathbf{P}\Big[\sup_{0\leq s\leq t}\Big|\int_0^s \int_{M(E)^\circ} \langle\nu,1\rangle 1_{\{\langle\nu,1\rangle> 1\}}\tilde{N}(\mathrm{d}r,\mathrm{d}\nu)\Big|\Big] \\
& + (\|\gamma\| + \|b\|)\mathbf{P}\Big[\int_0^t \langle Z_{j,k}(s),1\rangle \mathrm{d}s\Big] \\
\leq\ & 2\Big\{\mathbf{P}\Big[\int_0^t \langle Z_{j,k}(s),c\rangle \mathrm{d}s\Big]\Big\}^{\frac{1}{2}} + \|\kappa\|\int_0^t \mathbf{P}\big[\langle\lambda,\rho_s\rangle\big]\mathrm{d}s \\
& + \Big\{\mathbf{P}\Big[\int_0^t \mathrm{d}s \int_E Z_{j,k}(s,\mathrm{d}x)\int_{\{\langle\nu,1\rangle\leq 1\}} \langle\nu,1\rangle^2 H(x,\mathrm{d}\nu)\Big]\Big\}^{\frac{1}{2}} \\
& + 2\mathbf{P}\Big[\int_0^t \mathrm{d}s \int_E Z_{j,k}(s,\mathrm{d}x)\int_{\{\langle\nu,1\rangle> 1\}} \langle\nu,1\rangle H(x,\mathrm{d}\nu)\Big] \\
& + (\|\gamma\| + \|b\|)\mathbf{P}\Big[\int_0^t \langle Z_{j,k}(s),1\rangle \mathrm{d}s\Big],
\end{aligned}$$

where $\|\kappa\| = \|\kappa(\cdot,1)\|$."

- (Page 249, line –5) Replace (10.32) by

$$\text{“}\ \sup_{x\in E}\int_{M(E)^\circ}\langle\nu,1\rangle^2 H(x,\mathrm{d}\nu) + \sup_{y\in F_1}\int_{M(E)^\circ}\langle\nu,1\rangle^2 K(y,\mathrm{d}\nu) < \infty,\ \text{”}.$$