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Representation theory of Artin algebras, by Maurice Auslander, Idun Reiten, and Serre O. Smalø, Cambridge Stud. Adv. Math., vol. 36, Cambridge Univ. Press, 1995, xiv+423 pp., \$64.95, ISBN 0-521-41134-3

Here is the book you may have been waiting for for a long time, maybe for fifteen years: a general introduction to the new representation theory of finite-dimensional algebras. There has been a very surprising development in the last twenty-five years, and the need for an exposition which outlines all the techniques which have been found to be fruitful was felt by many mathematicians. The current representation theory is often referred to as the representation theory of quivers or as the Auslander-Reiten theory, stressing in this way two highlights of the new theory. One has to be grateful to the authors for a competent and readable introduction to the subject.

There do exist different approaches to the representation theory of algebras, and there have been fierce struggles about the relevance of various contributions. It is remarkable that the present book tries to keep away from all these fights; it gives a well-founded and balanced account, and even the historical references seem to be done very carefully. Of course, it is the Auslander-Reiten approach which serves as the guideline for the presentation, but other techniques are incorporated whenever this seemed to be suitable.

It may be worthwhile to review at least partly the present status of the subject before we look at the actual content of the book. What is an artin algebra Λ? It is an artin ring (a ring satisfying the descending chain condition on left ideals) with a "large" center; more precisely, there is given a commutative artin ring R, and one considers an R-algebra Λ which is of finite length when considered as R-module. For outlining the relevance of the subject, let me stick to the special case where $R = k$ is a field; thus we consider finite-dimensional k-algebras (a typical other choice for R would be a ring of the form $\mathbb{Z}/p^n$, where p is a prime number and $n \geq 2$; artin $\mathbb{Z}/p^n$-algebras are of interest for example in number theory, but also elsewhere). Some concepts such as duality are easier to understand in case R is a field, and there should be no difficulty to visualize afterwards the general situation.

Thus, let k be a field and Λ a finite-dimensional k-algebra (algebras are always assumed to be associative and with 1). We consider finite-dimensional representations of Λ; these are k-algebra homomorphisms from Λ into the endomorphism algebra of some finite-dimensional k-space V. Equivalently, such a homomorphism $\phi : \Lambda \to \operatorname{End}_k(V)$ makes V into a left Λ-module: this means that one defines a scalar multiplication $av = \phi(a)(v)$ for $a \in \Lambda, v \in V$; thus one considers instead of ϕ a corresponding map $\Lambda \times V \to V$ (the fact that ϕ is an algebra homomorphism can be reformulated in terms of this scalar multiplication, and one gets axioms of the form $(a_1a_2)v = a_1(a_2v)$ and so on). Of importance is the notion of a direct sum: given two Λ-modules M_1, M_2, one forms the direct sum $M_1 \oplus M_2$ of the underlying vector spaces and defines $a(m_1, m_2) = (am_1, am_2)$. Non-zero modules which cannot be written as the direct sum of two proper submodules are said to

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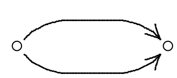
be *indecomposable.* Of course, any finite-dimensional module can be written as a direct sum of indecomposable modules, and there is an old result called the theorem of Krull-Remak-Schmidt which asserts that such a decomposition is unique up to isomorphism. This means that for many questions one may restrict one's self to the consideration of indecomposable modules. One should be careful and distinguish between indecomposability and irreducibility: A module M is said to be irreducible (or, as it is now more common, to be *simple*) provided $M \neq 0$ and M has besides 0 and M no other submodules. A simple module is indecomposable, but an arbitrary indecomposable module may have plenty of submodules (what is excluded are proper submodules M_1, M_2 which have zero intersection and which together generate the module). For some well-known algebras Λ, all indecomposable modules are simple: for example, Maschke's theorem asserts this for the group algebra of a finite group over a field k provided the characteristic of k does not divide the order of the group. Such algebras are said to be semisimple. The representation theory of algebras focuses attention on algebras which are not semisimple!

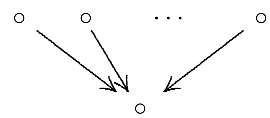
The first aim of the representation theory is to get information about the possible structure of indecomposable modules. One is looking for invariants which distinguish the isomorphism classes but also for algorithms in order to construct suitable indecomposable modules. A final aim may be the complete description of all indecomposable modules up to isomorphism, but this seems to be hard to achieve for most of the algebras, since they have what one calls wild representation type. At least in the case when one deals with an algebraically closed base field k, it was conjectured by Donovan and Freislich and established by Drozd that there is a trichotomy between finite, tame and wild representation type: An artin algebra Λ is said to be of *finite representation type,* provided there are only finitely many isomorphism classes of indecomposable Λ-modules. Only in this case, there is a bound on the length of the indecomposable Λ-module: this was asserted by the first Brauer-Thrall conjecture and was established by Roiter in 1968. It is one of the themes considered in the book under review. One expects that any artin algebra which is not of finite representation type will have families of pairwise non-isomorphic indecomposable modules indexed by projective lines. The corresponding second Brauer-Thrall conjecture was verified by Bautista and Bongartz in the case of an algebraically closed base field, but these investigations are based on many technical considerations and therefore are outside the scope of the book.

One should be aware that there are plenty of examples of finite-dimensional algebras which arise in other parts of algebra, but also in geometry and even in analysis. There are the group algebras of finite groups, there are semigroup algebras, there are the incidence algebras of posets. The Kronecker algebra $A = \begin{pmatrix} k & k^2 \\ 0 & k \end{pmatrix}$ is a four-dimensional algebra, and the A-modules can be identified with the representations of the quiver



thus with pairs (α, β) of linear maps $\alpha, \beta : V \to W$, where V, W are k-spaces, or, after choosing bases of V and W, with pairs of matrices of the same size: such pairs are sometimes called matrix pencils. The classification of the indecomposable matrix pencils is of importance, for example, for solving differential equations. The problem of classifying the indecomposable A-modules was considered by Weierstraß and then solved by Kronecker in 1890 (in fact, they considered the equivalent problem of classifying pairs of symmetric bilinear forms).

The next example to be presented are the $B(n)$-modules, where $B(n)$ is the ring of all $(n+1)\times(n+1)$-matrices over k, with non-zero entries allowed only on the main diagonal and in the last row. The $B(n)$-modules are just the representations of the following quiver:



The algebra $B(n)$ has $n+1$ simple modules, one of them being projective. The $B(n)$-modules which do not split off a non-projective simple module (or, equivalently, the representations of the quiver using only injective maps) may be identified with the $(n+1)$-tuples $(V_1, \dots, V_{n+1})$ where $V_1, \dots, V_n$ are subspaces of a k-space V_{n+1}. The problem of classifying $B(n)$-modules just means classifying the possible mutual position of n subspaces in a k-space. For example, for $n = 2$, we deal with two subspaces V_1, V_2 of a k-space V_3, and we know from elementary linear algebra that there exists a basis of V_3 which is compatible both with V_1 and V_2 (take a basis of $V_1 \cap V_2$, extend it to a basis of V_1 and to a basis of V_2; in this way we obtain a basis of $V_1 + V_2$, and now we extend this to a basis of V_3). For $n = 3$, there is already an obstacle: consider $W_4 = k^2$ with the subspaces $W_1 = k \times 0$, $W_2 = 0 \times k$, and $W_3 = \{(x, x) | x \in k\}$; clearly, no basis of W_4 will be compatible with all three subspaces. However, it is easy to see that for $n = 3$, this is the only difficulty; any k-space with three subspaces may be written as the direct sum of one-dimensional subspaces and copies of $W = (W_1, W_2, W_3, W_4)$ such that this decomposition is compatible with the three subspaces. In particular, $B(3)$ is of finite representation type. For $n = 4$, there are infinitely many isomorphism classes of indecomposable $B(n)$-modules; a complete classification has been exhibited by Gelfand and Ponomarev in 1970. Note that in the case $n = 4$, the underlying graph of the given quiver is usually labelled $\widetilde{D}_4$; it is one of the so-called Euclidean diagrams, whereas for $n = 3$, one deals with the Dynkin diagram D_4.

Another source of examples is the representation theory of Lie algebras. Starting with a semisimple finite-dimensional complex Lie algebra $\mathbf{g}$, there is the famous category $\mathcal{O}$ introduced by Berstein-Gelfand-Gelfand in order to deal with highest weight modules such as the Verma modules. The category $\mathcal{O}$ contains all finite-dimensional representations of $\mathbf{g}$, but most of the representations belonging to $\mathcal{O}$ are infinite-dimensional. The category $\mathcal{O}$ decomposes into blocks $\mathcal{O}_\lambda$, and each block has only finitely many simple objects. Such a block is equivalent to the module category $\mod C(\lambda)$ of a finite-dimensional $\mathbb{C}$-algebra $C(\lambda)$. Namely, $\mathcal{O}_\lambda$ is, first, an abelian category with only finitely many simple objects; second, any object in $\mathcal{O}_\lambda$ has finite length and there is a bound on the Loewy length of the objects; and, third, the Hom- and Ext^1-groups are finite-dimensional $\mathbb{C}$-spaces. These three properties characterize the categories which are equivalent to the module category of a finite-dimensional $\mathbb{C}$-algebra. In order to display properties of arbitrary objects of $\mathcal{O}_\lambda$, thus properties of (usually infinite-dimensional) representations of $\mathbf{g}$, we can use the equivalent category $\mod C(\lambda)$, thus dealing only with finite-dimensional vector spaces. Since the category $\mathcal{O}$ is a highest-weight category, the algebras $C(\lambda)$

have an additional structure: they are quasi-hereditary, and this implies that we deal with algebras $C(\lambda)$ of finite global dimension.

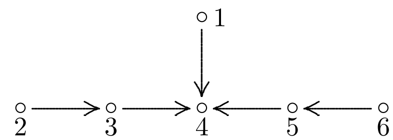
Highest-weight categories or, equivalently, the module categories of quasi-hereditary algebras, have successfully been studied by looking at the corresponding derived categories. Derived categories of module categories are presently one of the main points of interest: after all, several abelian categories arising in analysis, in topology and in algebraic geometry (the categories of coherent sheaves over suitable projective varieties as well as categories of what are said to be perverse sheaves) turn out to be derived equivalent to module categories. But derived categories are also the natural setting for many problems inside the representation theory itself; the use of tilting modules and tilting functors should always be interpreted in this way.

Clearly, for a finite-dimensional k-algebra Λ, all the finitely generated Λ-modules are finite-dimensional and therefore of finite length: Given such a module M, there is a sequence $0 = M_0 \subset M_1 \subset \cdots \subset M_t$ of submodules such that all the factors M_i/M_{i-1} are simple (they are called the composition factors of M). For any simple module S the multiplicity of S occurring as a composition factor is an invariant of the module; it is called the Jordan-Hölder multiplicity of S in M. Note that there are only finitely many isomorphism classes of simple Λ-modules, say $S_1, \dots, S_n$, and we denote by $\mathbf{dim}\, M$ the dimension vector of M: this is the function which attaches to the simple module S_j the Jordan-Hölder-multiplicity $(\mathbf{dim}\, M)_j$ of S_j in M. We may consider the dimension vector $\mathbf{dim}\, M$ of M as the element of the Grothendieck group $K_0(\Lambda)$ corresponding to M. Here, $K_0(\Lambda)$ is the Grothendieck group of all (finitely generated) Λ-modules modulo exact sequences. The Jordan-Hölder theorem just asserts that inside the Grothendieck group the module M is identified with the formal sum of its compositon factors. There is the following straightforward question: which indecomposable modules are determined up to isomorphism by their dimension vectors? This question is one of the central motifs of the Auslander-Reiten-Smalø book. It has turned out to be very fruitful to endow $K_0(\Lambda)$ with additional structures in order to be able to recover the dimension vectors of the indecomposable modules. First of all, $K_0(\Lambda)$ has a distinguished basis, given by the dimension vectors of the simple modules, and thus there is a notion of positivity. Second, in case we deal with algebras of finite global dimension, in particular for hereditary algebras (these are the algebras of global dimension at most 1), we may define an integer-valued function q on dimension vectors by

$$q(\mathbf{dim}\, M) = \sum_{i \geq 0} (-1)^i \dim_k \operatorname{Ext}^i_\Lambda(M, M);$$

this is well-defined and extends to a quadratic form on all of $K_0(\Lambda)$; it is called the homological quadratic form of Λ.

The use of quadratic forms in representation theory has produced exciting results. The first result of this kind is due to Gabriel: he has shown that a hereditary k-algebra Λ, where k is an algebraically closed field, is of finite representation type if and only if the homological quadratic form q_Λ on $K_0(\Lambda)$ is positive definite and that in this case $\mathbf{dim}$ furnishes a bijection between the isomorphism classes of the indecomposable Λ-modules and the positive roots of q_Λ. The quadratic forms encountered by Gabriel are those labelled A_n, D_n, E_6, E_7, E_8 in Lie theory. If one removes the restriction that k is algebraically closed, then one obtains also the missing cases B_n, C_n, F_4, G_2. Note that in this setting one has to adopt a convention from Lie theory concerning the notion of a root; for example, for the simply laced cases a vector x is called a root provided it satisfies $q_\Lambda(x) = 1$ (after all, we deal with a positive definite form, so there are no non-trivial vectors x with $q_\Lambda(x) = 0$). A typical example of a hereditary algebra Λ of type E_6 is the path algebra of the quiver Δ



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the quiver is obtained from the Dynkin diagram E_6 by endowing the edges with an orientation, and any choice of orientation is allowed. The orientation displayed above is called the subspace orientation, since all arrows point to the central vertex. Here, the interesting representations are those where all maps involved are inclusion maps; thus one deals with a vector space V_4 (attached to the central vertex), and five subspaces V_1, V_2, V_3, V_5, V_6 of V_4, with $V_2 \subseteq V_3$ and $V_6 \subseteq V_5$. The corresponding quadratic form is

$$\sum_{i=1}^{6} X_i^2 - X_1X_4 - \sum_{i=2}^{5} X_iX_{i+1}.$$

It is well-known and easy to see that this quadratic form has a unique maximal root; we want to display it in the shape of the quiver:

$$\begin{matrix} & & 2 & & \\ 1 & 2 & 3 & 2 & 1 \end{matrix}.$$

The bijection between the positive roots and the indecomposable Λ-modules assures that there is (up to isomorphism) a unique representation M of the quiver Δ having this dimension vector. One can show that any indecomposable Λ-module occurs as a subquotient of M, in particular, M has to be faithful.

The bijection between the indecomposable Λ-modules, where Λ is a hereditary artin algebra of Dynkin type Δ, and the positive roots of the corresponding quadratic form has a deeper reason: If we assume that our base field k is finite, then the free abelian group with basis the set of isomorphism classes of Λ-modules can be made into an associative algebra, the product being defined by counting the number of suitable filtrations of modules. The (twisted generic) Hall algebra which one obtains in this way is just the Drinfeld-Jimbo quantization of the positive part of the Lie algebra of type Δ. The hereditary artin algebras are an interesting starting point for constructing quantum groups, and the quantum Serre relations occur as universal relations for dealing with the possible composition series of modules.

The use of quivers, integral quadratic forms, and root systems, but also similar considerations invoking posets and integral bilinear forms, give a strong combinatorial flavour to the representation theory. At least for algebras of finite representation type, the invariants of the indecomposable modules are discrete ones, and the given base field usually will play no essential role. Also for a representation infinite algebra, part of the structure theory of the module category in question will rely on combinatorial data, but in addition one will have to use methods from algebraic geometry. Until now, only a few cases have been studied carefully.

Of particular interest is the Auslander-Reiten quiver $\Gamma(\Lambda)$ of Λ. It is defined for any artin algebra Λ and is a locally finite quiver whose vertices are just the isomorphism classes of the indecomposable Λ-modules. The components of $\Gamma(\Lambda)$ are usually infinite, the only exception occurs (necessarily) for algebras of finite representation type. This result due to Auslander was the starting point of many investigations. After all, in this way any indecomposable Λ-module M is related usually to countably many other ones, and the position of these modules inside the module category describes the interrelation of M with the remaining modules.

Let me return to the algebras of finite representation type, since they are in the center of the representation theory as described by Auslander-Reiten-Smalø. An effective study of the corresponding module category will use the covering theory as introduced by Gabriel and his school, and also by Gordon and Green, in order to reduce the investigation to representation directed algebras. For representation directed algebras, the construction of preprojective components provides a convenient method for obtaining all the indecomposable modules. In this way, the complete module category is displayed, and it seems that all standard questions (for example, concerning the homological behaviour or possible degenerations) can be answered without problems. The indecomposable modules over a representation directed algebra can be related to corresponding modules over hereditary algebras using tilting theory. The possibility of reducing problems about algebras of finite representation type via covering theory and tilting theory to those dealing with hereditary algebras shows the importance of the representation theory of hereditary algebras, and this is a topic which is (at least partly) covered in the book by Auslander, Reiten and Smalø.

Let us now turn our attention to the various chapters of the book. I have tried to outline the broad scope of the present theory, but a textbook which wants to start with first principles has to be more modest. The only application which is treated in detail is the modular representation theory of groups, but one should not expect to find references to something like quantum groups. Also, there are no quasi-hereditary algebras and no perverse sheaves: the use of derived categories is not touched at all. The authors concentrate on the internal structure theory for artin algebras and their module categories, and this they do very well.

The general setting of the book is the following: there is given a fixed commutative artin ring R, and one investigates an artin R-algebra Λ and the (finitely generated left) Λ-modules. Now Λ usually will be non-commutative; thus one may be tempted to work also with right Λ-modules. Instead of doing so, the authors consider besides Λ also the opposite ring Λ^{op}; note that the right Λ-modules are just left Λ^{op}-modules. Since one deals with an R-algebra Λ, there exists a duality between the category of (finitely generated left) Λ-modules and the category of (finitely generated left) Λ^{op}-modules.

The first two chapters of the book are of an introductory nature, on artin rings and on artin algebras: in particular, the structure of the projective and of the injective modules is explained. Of particular interest is a section called *projectivization*; it deals with the relationship between a module M and its endomorphism ring E. Many properties of M can be read off from properties of E. Note that the indecomposable direct summands of M correspond bijectively to the indecomposable projective E-modules; the bijection is furnished by the functor $\text{Hom}_\Lambda(M, -)$. This seems to be the natural framework for many constructions in representation theory; in particular, the Morita equivalences arise in this way.

Chapter III is devoted to the exhibition of important classes of examples: First of all, the path algebras of finite quivers are introduced. It is shown that the representations of a quiver are nothing else than the modules over the corresponding path algebras (III.1.5). Recall that for any artin algebra Λ, the factor algebra $\overline{\Lambda}$ modulo the radical is a product of matrix rings over division rings, and up to Morita equivalence, one may assume that $\overline{\Lambda}$ is actually a product of division rings. In case Λ is a k-algebra and $\overline{\Lambda}$ is actually a product of copies of k, the authors call Λ an elementary algebra. Note that any elementary algebra can be written as a factor algebra of the path algebra of a finite quiver. In this chapter also, group algebras are introduced and basic properties such as Maschke's theorem are derived. In later parts, there are frequent references to group algebras: the authors present the characterization of those of finite representation type: the group algebra kG is of finite representation type if and only if the p-Sylow groups of G are cyclic, where p is the characteristic of k, and they give the precise structure of the corresponding blocks.

Chapters IV, V, and VII may be considered as the heart of the book. Here, the existence of almost split sequences is shown, and related notions such as the dual of the transpose of a module and that of an irreducible map are discussed. For any artin algebra Λ, the object to be considered is its Auslander-Reiten quiver: as we have mentioned, its vertices are the isomorphism classes of the indecomposable Λ-modules, and one draws an arrow from the isomorphism class of X to the isomorphism class of Y provided there exists an irreducible map $X \to Y$. What one obtains in this way is a locally finite quiver, usually with infinitely many components, and it is the structure of this Auslander-Reiten quiver which is of main concern.

In between, there is Chapter VI dealing with artin algebras of finite representation type. Maybe the authors want to stress that a large part of the theory of artin algebras of finite representation type can be presented without the explicit notion of the Auslander-Reiten quiver. However, in this way there is some cumbersome repetition and actually some awkward duplication of notions: they introduce in this chapter the notion of a *component of* $\operatorname{ind}\Lambda$ for what later are called the *connected components* of the Auslander-Reiten quiver of Λ (what should be non-connected components?). Three main results in Chapter VI have to be singled out: First of all (VI.1.4), if the Auslander-Reiten quiver of a connected artin algebra Λ has a component with a bound on the length of the modules in the component, then Λ is of finite representation type and such an artin algebra has just one component. In this way, Auslander has strengthened the assertion of the first Brauer-Thrall conjecture as proved before by Roiter: an artin algebra of bounded representation type is of finite representation type. The proof given in the book follows Yamagata and is based on the Harada-Sai Lemma (and the existence of almost split sequences). Second, given an artin algebra Λ of finite representation type, say, with $M_1, \dots, M_m$ being a complete list of indecomposable Λ-modules, then the endomorphism ring of the direct sum $\bigoplus_{i=1}^m M_i$ is what now is called an Auslander algebra: it has global dimension at most 2 and dominant dimension at least 2. Actually (VI.5.7) one obtains in this way a bijection between the Morita equivalence classes of artin algebras of finite representation type and Morita equivalence classes of Auslander algebras. The third result to be mentioned (VI.4.2) is a result for general artin algebras, not being restricted to those of finite representation type: Two modules M_1, M_2 are isomorphic, in case the R-modules $\operatorname{Hom}_\Lambda(N, M_1)$ and $\operatorname{Hom}_\Lambda(N, M_2)$ have the same length, for any Λ-module N, and also in case the R-modules $\operatorname{Hom}_\Lambda(M_1, N)$ and $\operatorname{Hom}_\Lambda(M_2, N)$ have the same length, for any Λ-module N.

Chapter VIII deals with hereditary algebras: first, the preprojective and the preinjective modules are constructed starting with the indecomposable projective or the indecomposable injective modules, respectively, and using the Auslander-Reiten translation. These are directing modules; thus they are uniquely determined by their dimension vectors. In case we deal with a hereditary algebra of finite representation type, all indecomposable modules are preprojective and also preinjective. The hereditary algebras of finite representation type can be characterized by the positivity of the homological quadratic form (VII.3.6), or equivalently (VII.5.4), by the fact that one deals with a disjoint union of Dynkin diagrams. For a hereditary algebra which is not of finite representation type, no indecomposable module is both preprojective and preinjective, and there are additional components, the regular ones. It is shown (VII.4.15) that the regular components are either tubes or of the form $\mathbb{Z}A_\infty$. The only representation infinite algebra which is discussed in detail is the Kronecker algebra; here the full classification of all indecomposables is given (VII.7.5).

Chapter IX presents properties of directing modules or, more generally, of indecomposable modules which do not belong to what are called short cycles. A short cycle is a pair of indecomposable modules M_1, M_2 such that there are maps $M_1 \to M_2$ and $M_2 \to M_1$ which are non-zero and non-invertible. Let M be indecomposable, and assume that it does not belong to a short cycle. (1) If M is not faithful, then M is annihilated by some non-zero idempotent (thus not all simple Λ-modules occur as composition factors of M). (2) If N is indecomposable and $\mathbf{dim}\, M = \mathbf{dim}\, N$, then M and N are isomorphic. (3) If N is indecomposable and M and N have isomorphic top and isomorphic socle, then M and N are isomorphic.

Chapter X is devoted to stable equivalence: two artin algebras Λ and Λ' are said to be stably equivalent provided the module categories become equivalent after factoring out all the maps which factor through projective modules. Stable equivalence plays an important role in the modular representation theory of finite groups: the blocks of finite representation type are known to be stably equivalent to serial algebras (in the book, serial algebras are called Nakayama algebras), and the book shows that any symmetric artin algebra which is stably equivalent to a serial algebra is given by a Brauer tree (X.3.14). But the basic example of stably equivalent artin algebras seems to be the following: Let Λ be an artin algebra with radical square zero. Then there exists a hereditary artin algebra Λ' which is stably equivalent to Λ (X.2.4). In particular, one may decide the representation type of Λ by considering its "separated quiver" (X.2.6). It is a pity that the authors did not include the local version of this separation procedure, the process of removing "nodes" as considered by Martinez.

The last chapter, XI, deals with morphisms which are determined by modules. This general theory introduced by Auslander in his Philadelphia notes was one of his main concerns. He always stressed the importance of this theory, which unifies many different considerations. So it seems natural that it has been chosen as the final topic of the book.

As prerequisites, the authors require only some basic notions in ring and module theory as well as some homological algebra. The requirements from ring theory include the structure of semisimple artin rings; from module theory one should be familiar with the definitions of projective, injective and semisimple modules. The authors recall most of the basic facts; they even provide proofs of the theorems of Jordan-Hölder and of Krull-Remak-Schmidt. (Actually, it seems strange that the last result is presented only in the narrow setting of finitely generated modules over artin algebras.) From homological algebra, the definition of Ext^1 and Ext^2 and their properties are used; there is a special section outlining the relationship between Ext^1 and equivalence classes of exact sequences. Another introductory section deals with additive categories, or better, R-categories; it contains the proof that a functor is an equivalence if and only if it is full, faithful and dense. But it should be noted that the authors are quite reluctant to use functorial considerations; in particular they avoid the use of functor categories.

One observes that the theory presented in the book dates back to the seventies. There are very few results in the book which were not known by 1978, but such a polished way of presentation as is given in the book may not have been possible at that time. It is very important that these topics, which are at the basis of all the further developments, are now available in a form which is readable and accessible also for students. It seems to the reviewer that the book may have gained if the general construction of preprojective components, as well as the structure of the module category of a tame hereditary algebra, would have been included. These are essential working tools which would fit well into the concept of the book (and were known by 1978). In addition, we feel that tilting theory and covering theory belong to the basic methods in representation theory. But these are just minor complaints. I should add that there are plenty of exercises which will be helpful for any reader: they serve as an illustration of the theory and present additional material. Of particular interest is the list of eleven open problems. The book can be recommended without reservation! It surely will serve as a standard reference.

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