A martingale transformation for superprocesses*

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Abstract

In this note, we prove a new martingale transformation for superprocesses with general branching mechanisms which could be regarded as a Girsanov's type transformation in measure-valued setting. As an application, a criterion for global extinction of superprocesses with spatially dependent branching mechanisms is given.

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1 Introduction and main results

A Girsanov's type theorem for superprocesses was first proved by Dawson (1978). A bivariate version was given by Evans and Perkins (1994); see also Theorem IV.1.6 in Perkins (2002). Motivated by some recent works on Girsanov's type theorems for continuous state branching processes and Lévy trees; see Abraham and Delmas (2010), in this work, we will prove a Girsanov's type theorem for superprocesses with general branching mechanisms. The processes could be discontinuous and have non-local branchings.

Suppose that E is a Lusin topological space and d is a metric for its topology so that the d-completion of E is compact. Let $\xi = (\Omega, \mathcal{G}, \mathcal{G}_t, \xi_t, \mathbf{P}_x)$ be a conservative Borel right process in E with transition semigroup $(P_t)_{t\geq 0}$. We denote by $B(E)$ the Banach space of bounded Borel functions on E endowed with the supremum/uniform norm "$\|\cdot\|$". Let $C(E)$ be the subspace of $B(E)$ consisting of continuous functions. We use the superscript "+" to denote the subset of positive elements of the function spaces, e.g., $B^+(E)$ and $C^+(E)$. For a measure μ and a Borel function f on E write $\mu(f)$ or $\langle \mu, f\rangle$ for $\int f d\mu$ if the integral exists. Let $M(E)$ denote the space of finite measures on E endowed with the topology of weak convergence.

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To describe our superprocesses, we first define an operator $f \mapsto \psi(\cdot, f)$ from $B(E)^+$ to $B(E)$ as follows. Let $b \in B(E)$ and $c \in B(E)^+$. Let $\eta(x, dy)$ be a bounded kernel on E and $H(x, d\nu)$ a σ-finite kernel from E to $M(E)^o := M(E)\backslash\{0\}$, where 0 is the null measure. Suppose that

$$\sup_{x\in E} \int_{M(E)^o} \left[\nu(1) \wedge \nu(1)^2 + \nu_x(1)\right] H(x, d\nu) < \infty, \tag{1.1}$$

where $\nu_x(dy)$ denotes the restriction of $\nu(dy)$ to $E\backslash\{x\}$. For $x \in E$ and $f \in B(E)^+$ write

$$\begin{aligned}\psi(x, f) = b(x)f(x) + c(x)f^2(x) - \int_E f(y)\eta(x, dy) \\ + \int_{M(E)^o} \left[e^{-\nu(f)} - 1 + \nu(\{x\})f(x)\right] H(x, d\nu).\end{aligned} \tag{1.2}$$

Let $(\mathcal{A}, D(\mathcal{A}))$ denote the weak generator of $(P_t)_{t\geq 0}$, i.e., for $f \in D(\mathcal{A})$ and every $x \in E$,

$$f(\xi_t) - f(x) - \int_0^t \mathcal{A}f(\xi_s)ds, \quad t \geq 0 \tag{1.3}$$

is a $\mathcal{G}_t$-martingale under $\mathbf{P}_x$. We further assume that $P_tC(E) \subset C(E)$ and $D(\mathcal{A}) \subset C(E)$.

Definition 1.1 *We call an $M(E)$-valued Markov process the (ξ, ψ)-superprocess if its transition semigroup $(Q_t)_{t\geq 0}$ on $M(E)$ is given by*

$$\int_{M(E)} e^{-\nu(f)} Q_t(\mu, d\nu) = \exp\{-\mu(V_t f)\}, \quad f \in B(E)^+, \tag{1.4}$$

where $(t, x) \mapsto V_t(x, f)$ is the unique locally bounded positive solution to

$$V_t(x) = P_t f(x) - \int_0^t ds \int_E \psi(y, V_s) P_{t-s}(x, dy), \quad t \geq 0, x \in E. \tag{1.5}$$

The ξ and ψ will be refereed to as the underlying motion and the branching mechanism of the (ξ, ψ)-superprocesses. More details could be found in Section 2.3 of Li (2011).

Let $\mathcal{X}$ be the set of all $M(E)$-valued right continuous functions on $[0, \infty)$. Let $\mathcal{F}$ be the σ-field generated by all finite dimensional cylinder sets in $\mathcal{X}$. We denote by $X = \{X_t : t \geq 0\}$ the coordinate process on $\mathcal{X}$ and let $\{\mathcal{F}_t : t \geq 0\}$ be its natural filtration. By Theorem 5.12 in Li (2011), the (ξ, ψ)-superprocess has a right realization in $M(E)$. Thus, for $\mu \in M(E)$, we have a probability measure $\mathbf{P}^\psi_\mu$ on $(\mathcal{X}, \mathcal{F})$ such that under $\mathbf{P}^\psi_\mu$, $X = \{X_t : t \geq 0\}$ is a (ξ, ψ)-superprocess and $\mathbf{P}^\psi_\mu[X_0 = \mu] = 1$.

For $q \in B(E)$, assume

$$\sup_{x\in E} \int_{\{\nu:\nu(1)>1\}} \left[\nu(1) + \nu_x(1)\right] e^{-\nu(q)} H(x, d\nu) < \infty. \tag{1.6}$$

Then

$$\psi_q(x, f) := \psi(x, q + f) - \psi(x, q)$$

$$= \left[b(x) + 2c(x)q(x) + \int_{M(E)^o} \nu(\{x\})(1 - e^{-\nu(q)})H(x, d\nu)\right] f(x)$$
$$+c(x)f^2(x) - \int_E f(y)\eta(x, dy)$$
$$+ \int_{M(E)^o} \left[e^{-\nu(f)} - 1 + \nu(\{x\})f(x)\right] e^{-\nu(q)} H(x, d\nu)$$

is well-defined and thus it is also a branching mechanism. For $q \in D(\mathcal{A})$ and $t \geq 0$, define

$$H_t^q = \exp\left\{\mu(q) - X_t(q) + \int_0^t X_s(\mathcal{A}q - \psi(\cdot, q))ds\right\}. \tag{1.7}$$

According to Proposition 2.27 and Lemma 2.17 in Li (2011) we have that, under $\mathbf{P}_\mu^\psi$, for any $0 \leq t \leq T$,

$$X_t(1) < \infty \quad \text{and} \quad \int_0^t X_s(1)ds < \infty.$$

Thus if $q \in D(\mathcal{A})^+$ and $\psi(\cdot, q) - \mathcal{A}q \in B^+(E)$, then $H_t^q > 0$ for $0 \leq t \leq T$.

Theorem 1.1 *For $q \in D(\mathcal{A})$, assume that (1.6) holds. If*

$$\lim_{l\to\infty} \inf_{x\in E} \psi(x, l) = +\infty, \tag{1.8}$$

then

i) under $\mathbf{P}_\mu^\psi$, $\{H_t^q : t \geq 0\}$ is an $\mathcal{F}_t$-martingale;

ii) for each $t \geq 0$,

$$\frac{d\mathbf{P}_\mu^{\psi_q}|_{\mathcal{F}_t}}{d\mathbf{P}_\mu^\psi|_{\mathcal{F}_t}} = H_t^q. \tag{1.9}$$

Remark 1.1 *(1.8) holds if $\inf_{x\in E} c(x) > 0$ or $\inf_{x\in E} \int_{M(E)^o} \nu(1)H(x, d\nu) = +\infty$.*

Remark 1.2 *When ψ satisfies Conditions 7.1 and 7.2 in Li (2011) and q belongs to the domain of the strong generator of ξ, Theorem 7.13 in Li (2011) proved that $\{H_t^q : t \geq 0\}$ is a local martingale where it does not require (1.6) and (1.8).*

In the following, we make the convention that $\infty \cdot 0 = 0$. We give a criterion for global extinction of (ξ, ψ)-superprocesses.

Corollary 1.1 *Assume $q \in D(\mathcal{A})^+$ and $\psi(\cdot, q) - \mathcal{A}q \in B^+(E)$. Then, under $\mathbf{P}_\mu^\psi$, H^q converges a.s. to a limit H_∞^q such that*

$$H_\infty^q 1_{\{\tau_0<+\infty\}} = \exp\left\{\mu(q) + \int_0^\infty ds X_s(\mathcal{A}q - \psi(\cdot, q))\right\} 1_{\{\tau_0<+\infty\}},$$

where $\tau_0 = \inf\{t \geq 0 : X_t(1) = 0\}$. Furthermore, we have

$$\frac{d\mathbf{P}_\mu^{\psi_q}}{d\mathbf{P}_\mu^\psi} = H_\infty^q 1_{\{\tau_0<+\infty\}} \tag{1.10}$$

and

$$\mathbf{P}_\mu^\psi[\tau_0 < \infty] = \mathbf{P}_\mu^{\psi_q}\left[(H_\infty^q)^{-1} 1_{\{\tau_0<+\infty\}}\right]. \tag{1.11}$$

Remark 1.3 *When* $\psi(x,f) = b(x)f(x) + c(x)f(x)^2$, *for some very specialized* q, b *and* c, *(1.10) has been proved in Delmas and Hénard (2011).*

(1.11) means that if one wants to know the global extinction probability of (ξ, ψ)-superprocesses, we only need to calculate the r.h.s of (1.11). However, it is not so easy to know its exact value. In the next section, we will give an example to show how to make it easier in calculating the r.h.s. of (1.11) in some cases. The proofs of Theorem 1.1 and Corollary 1.1 will be given in Section 3.

2 An example

Recall that $\mathcal{A}$ denotes the weak generator of ξ. In this section, we will refer the (ξ, ψ)-superprocess as $(\mathcal{A}, \psi)$-superprocess for convenience. Let us consider a special case of $\psi(x,f) = b(x)f(x) + c(x)f(x)^2$ with $b \in C(E), c \in C^+(E)$. We further assume that $\inf_{x\in E} c(x) > 0$ and $1/c \in D(\mathcal{A})$. Define

$$X_t^c(dx) = \frac{1}{c(x)} X_t(dx), \quad t \geq 0.$$

Then by Corollary 3.6 in Delmas and Hénard (2011), $\{X_t^c : t \geq 0\}$ is a $(\mathcal{A}^c, \psi^c)$-superprocess under $\mathbf{P}_\mu^\psi$, where

$$\psi^c(x,f) = \left(b(x) - c(x)\mathcal{A}\left(\frac{1}{c}\right)\right) f(x) + f(x)^2 =: \tilde{b}(x) f(x) + f(x)^2$$

and $\mathcal{A}^c u = c(\mathcal{A}(u/c) - \mathcal{A}(1/c)u)$ for $u \in D_c(\mathcal{L}) := \{u \in C(E) : \frac{u}{c} \in D(\mathcal{A})\}$. Assume $\tilde{b} \in D(\mathcal{A}^c)$. Define

$$b_0 = \sup_{x\in E} \max\left(\tilde{b}(x), \sqrt{(\tilde{b}(x)^2 - 2\mathcal{A}^c\tilde{b}(x)) \vee 0}\right) \quad \text{and} \quad q = \frac{b_0 - \tilde{b}}{2}.$$

Then $q \in D(\mathcal{A}^c)^+$ and $\psi(\cdot, q) - \mathcal{A}^c q \in B^+(E)$. Furthermore, we have

$$\psi_q^c(x,f) = b_0 f(x) + f(x)^2.$$

Recall $\tau_0 = \inf\{t \geq 0 : X_t(1) = 0\}$.

Proposition 2.1 *Assume that* $\inf_{x\in E} c(x) > 0$ *and* $1/c \in D(\mathcal{A})$ *and* $\tilde{b} \in D(\mathcal{A}^c)$. *Then*

$$\mathbf{P}_\mu^\psi[\tau_0 < \infty] = \mathbf{P}_{\mu^c}^{\psi_q^c}\left[\exp\left\{-\mu^c(q) + \int_0^\infty ds X_s(\mathcal{A}^c q - b_0 q - q^2)\right\}\right], \tag{2.1}$$

where $\mu^c(dx) = \frac{1}{c(x)}\mu(dx)$.

Proof. Noting that ψ_q^c is a homogeneous branching mechanism and $b_0 > 0$, we have

$$\mathbf{P}_\mu^{\psi_q^c}[\tau_0 < \infty] = 1$$

for any $\mu \in M(E)$. Meanwhile, we also note that $\tau_0^c := \inf\{t \geq 0 : X_t^c(1) = 0\} = \tau_0$ and $\{X_t^c : t \geq 0\}$ is a $(\mathcal{A}^c, \psi^c)$-superprocess under $\mathbf{P}_\mu^\psi$. Thus

$$\mathbf{P}_\mu^\psi[\tau_0 < \infty] = \mathbf{P}_{\mu^c}^{\psi^c}[\tau_0 < \infty].$$

Then the desired result follows from (1.11). □

Remark 2.1 *Since ψ_q^c is a spatially homogeneous branching mechanism, it will be much easier to calculate the r.h.s of (2.1) whose value is just the global extinction probability of $(\mathcal{A},\psi)$-superprocess.*

3 Proofs

We first present a lemma on weighted occupation times of (ξ,ψ)-superprocesses, which will play an important role in the proof of Theorem 1.1.

Lemma 3.1 *[Theorem 5.15, Li (2011)] Suppose that $\lambda(ds)$ is a finite measure on $[0,t]$. Let $(s,x)\mapsto f_s(x)$ be a bounded positive Borel function on $[0,\infty)\times E$. Then we have*

$$\mathbf{E}_\mu^\psi \exp\left\{-\int_r^t X_s(f_s)\lambda(ds)\right\} = \exp\{-\mu(u_r)\},\quad 0\le r\le t, \tag{3.1}$$

where $(r,x)\mapsto u_r(x)$ is the unique bounded positive solution on $[0,t]\times E$ of

$$u_r(x) + \int_{[r,t]} \mathbf{P}_x[\psi(\xi_{s-r},u_{s-r})]ds = \int_{[r,t]} \mathbf{P}_x[f_s(\xi_{s-r})]\lambda(ds),\quad 0\le r\le t. \tag{3.2}$$

Proof of Theorem 1.1. **Step 1**: We first assume $q\in D(\mathcal{A})^+$ and $\psi(\cdot,q)-\mathcal{A}q\in B^+(E)$. We first show that $\mathbf{E}_\mu^\psi[H_t^q]=1$. By Lemma 3.1,

$$\mathbf{E}_\mu^\psi[H_t^q] = \exp\{\mu(q)-\mu(w_0^H)\}, \tag{3.3}$$

where $(r,x)\mapsto w_r^H(x)$ is the unique bounded positive solution on $[0,t]\times E$ of

$$\begin{aligned} w_r^H(x) &+ \int_{[r,t]} \mathbf{P}_x[\psi(\xi_{s-r},w_{s-r}^H)]ds \\ &= \mathbf{P}_x[q(\xi_{t-r})] + \int_{[r,t]} \mathbf{P}_x[\psi(\xi_{s-r},q)-\mathcal{A}q(\xi_{s-r})]ds. \end{aligned} \tag{3.4}$$

By (1.3), one can see $w_s^H = q$ for $0\le s\le t$. Then (3.3) gives $\mathbf{E}_\mu^\psi[H_t^q]=1$ which also implies the martingale property of $\{H_t^q : t\ge 0\}$.

Next we prove (ii). For any finite measure $\eta(ds)$ on $[0,t]$ and bounded positive function $g(s,x)$ on $[0,\infty)\times E$, again by Lemma 3.1, we have

$$\mathbf{E}_\mu^\psi\left[H_t^q \exp\left\{-\int_{[0,t]} X_s(g(s,\cdot))\eta(ds)\right\}\right] = \exp\{\mu(q)-\mu(w_0^g)\}, \tag{3.5}$$

where $(r,x)\mapsto w_r^g(x)$ is the unique bounded positive solution on $[0,t]\times E$ of

$$\begin{aligned} w_r^g(x) &+ \int_{[r,t]} \mathbf{P}_x[\psi(\xi_{s-r},w_{s-r}^g)]ds \\ &= \mathbf{P}_x[q(\xi_{t-r})] + \int_{[r,t]} \mathbf{P}_x[\psi(\xi_{s-r},q)-\mathcal{A}q(\xi_{s-r})]ds + \int_{[r,t]} \mathbf{P}_x[g(s,\xi_{s-r})]\eta(ds). \end{aligned} \tag{3.6}$$

Note that $w_r^g - q\in B^+(E)$. For $r\le t$ we set

$$w_r^{g,q}(x) = w_r^g(x) - q(x).$$

Then by (3.6)

$$
\begin{aligned}
& w_r^{g,q}(x)+\int_{[r,t]}\mathbf{P}_x[\psi_q(\xi_{s-r},w_{s-r}^{g,q})]ds \\
&\quad = w_r^{g,q}(x)+\int_{[r,t]}\mathbf{P}_x[\psi(\xi_{s-r},w_{s-r}^{g})]ds-\int_{[r,t]}\mathbf{P}_x[\psi(\xi_{s-r},q)]ds \\
&\quad = -q(x)+\mathbf{P}_x[q(\xi_{t-r})]-\int_{[r,t]}\mathbf{P}_x[\mathcal{A}q(\xi_{s-r})](ds)+\int_{[r,t]}\mathbf{P}_x[g(s,\xi_{s-r})]\eta(ds) \\
&\quad = \int_{[r,t]}\mathbf{P}_x[g(s,\xi_{s-r})]\eta(ds),
\end{aligned}
\tag{3.7}
$$

where the last equality follows from (1.3). Note that the equation

$$
v_r(x)+\int_{[r,t]}\mathbf{P}_x[\psi_q(\xi_{s-r},v_{s-r})]ds=\int_{[r,t]}\mathbf{P}_x[g(s,\xi_{s-r})]\eta(ds),\quad 0\le r\le t,
$$

also has unique positive bounded solution. Then by Lemma 3.1, (3.7) implies that

$$
\mathbf{E}_\mu^{\psi_q}\left[\exp\left\{-\int_{[0,t]}X_s(g(s,\cdot))\eta(ds)\right\}\right]=\exp\{-\mu(w_0^{g,q})\}=\exp\{\mu(q)-\mu(w_0^{g})\}. \tag{3.8}
$$

Thus by (3.5)

$$
\mathbf{E}_\mu^{\psi}\left[H_t^q\exp\left\{-\int_{[0,t]}X_s(g(s,\cdot))\eta(ds)\right\}\right]=\mathbf{E}_\mu^{\psi_q}\left[\exp\left\{-\int_{[0,t]}X_s(g(s,\cdot))\eta(ds)\right\}\right].
$$

Then, by monotone class theorem, for any non-negative $\mathcal{F}_t$-measurable random variable $\mathcal{W}$,

$$
\mathbf{E}_\mu^{\psi}[H_t^q\mathcal{W}]=\mathbf{E}_\mu^{\psi_q}[\mathcal{W}] \tag{3.9}
$$

which gives (1.9).

Step 2: Now, we consider the general case of $q\in D(\mathcal{A})$. With (1.6), ψ_q and $\psi(\cdot,q)$ are well-defined and ψ_q is a branching mechanism. By assumption (1.8), we may find a constant $c_q\ge ||q||$ such that $\psi(\cdot,c_q)-||\mathcal{A}q-\psi(\cdot,q)||\in B^+(E)$. Then by Step 1, for any non-negative $\mathcal{F}_t$-measurable random variable $\mathcal{W}$, $\mathbf{E}_\mu^{\psi}\left[H_t^{c_q}\mathcal{W}\right]=\mathbf{E}_\mu^{\psi_{c_q}}[\mathcal{W}]$. Noting that $H_t^{c_q}>0$, we also have

$$
\mathbf{E}_\mu^{\psi}[\mathcal{W}]=\mathbf{E}_\mu^{\psi_{c_q}}[(H_t^{c_q})^{-1}\mathcal{W}]. \tag{3.10}
$$

Meanwhile, since $\psi_q(\cdot,c_q-q)-\mathcal{A}(c_q-q)\in B^+(E)$ and $c_q-q\in D(\mathcal{A})^+$, by the results in Step 1, we have

$$
\begin{aligned}
& \mathbf{E}_\mu^{(\psi_q)_{c_q-q}}[\mathcal{W}] \\
&\quad = \mathbf{E}_\mu^{\psi_q}\left[\exp\left\{\mu(c_q-q)-X_t(c_q-q)+\int_0^t dsX_s(-\mathcal{A}q-\psi_q(\cdot,c_q-q))\right\}\mathcal{W}\right] \\
&\quad = \mathbf{E}_\mu^{\psi_q}[H_t^{c_q}(H_t^q)^{-1}\mathcal{W}],
\end{aligned}
\tag{3.11}
$$

where the last equality follows from $\psi_q(\cdot, c_q-q) = \psi(\cdot, c_q)-\psi(\cdot, q)$. Note that $(\psi_q)_{c_q-q}(x,f) = \psi_q(x, f+c_q-q) - \psi_q(x, c_q-q) = \psi_{c_q}(x,f)$. Since $H_t^q\mathcal{W}$ and $(H_t^{c_q})^{-1}H_t^q\mathcal{W}$ are also nonnegative $\mathcal{F}_t$ measurable random variables, by (3.10), (3.11),

$$\mathbf{E}_\mu^\psi[H_t^q\mathcal{W}] = \mathbf{E}_\mu^{\psi_{c_q}}[(H_t^{c_q})^{-1}H_t^q\mathcal{W}] = \mathbf{E}_\mu^{\psi_q}[\mathcal{W}], \tag{3.12}$$

which proves ii). Taking $\mathcal{W}=1$ in (3.12), we have that $\mathbf{E}_\mu^\psi[H_t^q] = \mathbf{E}_\mu^{\psi_q}[1] = 1$. Meanwhile, for $0 \le s \le t$ and any nonnegative $\mathcal{F}_s$-measurable random variable $\mathcal{W}$, applying (3.12) again yields

$$\mathbf{E}_\mu^\psi[H_t^q\mathcal{W}] = \mathbf{E}_\mu^{\psi_q}[\mathcal{W}] = \mathbf{E}_\mu^\psi[H_s^q\mathcal{W}]$$

which gives the martingale property of $\{H_t^q : t\ge 0\}$ under $\mathbf{P}_\mu^\psi$. We have completed the proof. □

Proof of Corollary 1.1. Since $q\in D(\mathcal{A})^+$ and $\psi(\cdot,q)-\mathcal{A}q\in B^+(E)$, we have $H_t^q \le e^{\mu(q)}$. Then H^q is a bounded nonnegative martingale under $\mathbf{P}_\mu^\psi$ and thus it converges to a limit H_∞^q a.s. as $t\to\infty$. By (3.9), for any nonnegative $\mathcal{F}$-measurable random variable $\mathcal{W}$,

$$\mathbf{E}_\mu^\psi[H_\infty^q\mathcal{W}] = \mathbf{E}_\mu^{\psi_q}[\mathcal{W}]. \tag{3.13}$$

On $\{\tau_0<\infty\}$, it is obvious that $H_\infty^q = e^{\mu(q)+\int_0^\infty ds X_s(\mathcal{A}q-\psi(\cdot,q))}$. Then (1.11) follows from the fact that $(H_\infty^q)^{-1}1_{\{\tau_0<+\infty\}}$ is also an $\mathcal{F}$-measurable random variable. We have completed the proof. □

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