Vol. XXX No. 2 SCIENTIA SINICA (Series A) February 1987

EXISTENCE THEOREMS FOR INTERACTING PARTICLE SYSTEMS WITH NON-COMPACT STATE SPACES*

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Received February 15, 1985.

ABSTRACT

For interacting particle systems, two existence theorems are obtained; each of them is described by two simpler conditions. The theorems cover the models treated in Refs. [1—8] and [15—17].

I. INTRODUCTION

An earlier and simpler model of interacting particle systems with the non-compact space——zero range process, was proposed by Spitzer[1] in 1970, for which, an existence theorem in a special case was given by Holley[2] (1970); and then, in general case, given by Liggett[3] (1973). Recently, Andjel[4] (1982) simplified the proof of [3] by using the technique developed in [5] (1981), where Liggett and Spitzer considered the existence and ergodicity for six models. On the other hand, Basis[6] (1976) studied quite a general model of interacting particle systems. Unfortunately, the results in [5] and [6] are not entirely suitable for the reaction-diffusion processes discussed in [7] and [8] which often come across in the study of nonequilibrium statistical physics (see Refs. [9] and [10]). The main difficulty is that the state space is neither locally compact nor σ-compact, and the operators usually are not locally bounded either. In this paper, we use the theory of q-processes, and the results in [11] combining with the idea in [5] and [6] to provide two existence theorems for interacting particle systems.

This paper rests on a basis of [11]. For its stationarity and ergodicity of the processes, we will discuss in another paper.

Throughout the paper, we suppose that S is a countable set. For each $u \in S$, let $(E_u, \rho_u, \mathscr{E}_u)$ be a complete separable metric space, where $\mathscr{E}_u$ is the algebra generated by the metric ρ_u. $(E, \mathscr{E})$ denotes the usual topological product space of $(E_u, \mathscr{E}_u)$ $(u \in S)$. Choose an arbitrary reference point $\theta = (\theta_u;\ u \in S)$, and suppose that $\{k_u\}$ in S is a positive summable sequence.

* Partially supported by the Ministry of Education and the Foundation of Advanced Research Centre in Zhongshan University.

For $x=(x_u;\ u\in S)$, $y=(y_u;\ u\in S)\in E$, and $\alpha\subset S$, define

$$p_\alpha(x,\ y)=\sum_{u\in\alpha}\rho_u(x_u,\theta_u)k_u,$$

(we also write $\rho_u(x_u,\ \theta_u)$ as $\rho_u(x)$, and $p_\alpha(x,\theta)$ as $p_\alpha(x)$), and set $E^\alpha=\{x\in E;\ p_{S\setminus\alpha}(x,\ \theta)=0\}$. The σ-algebra $\mathscr{E}^\alpha$ is induced on E^α by the σ-algebra $\mathscr{E}$. The natural projection of x on E^α is defined by

$$p_\alpha(x^\alpha,\ x)+p_{S\setminus\alpha}(x^\alpha,\ \theta)=0.$$

Let us denote the set of all finite subsets of S by $\mathscr{S}_f$. We assume that there is a fixed sequence $\{\Lambda_n\}_1^\infty\subset\mathscr{S}_f$ such that $\Lambda_n\uparrow S$. For each $n\geqslant 1$, there is also a fixed regular q-pair $q_n(x)-q_n(x,.)$ on $(E^{\Lambda_n},\ \mathscr{E}^{\Lambda_n})$ (see [11], [12]). The problem we are going to study is to find a limit process of these q-processes (jump processes) $P_n(t,\ x,.)$ determined uniquely by the given q-pair $q_n(x)-q_n(x,.)$. To this end, let p be an $\mathscr{E}$-measurable function (may be valued $+\infty$) satisfying:

1° $0\leqslant p(x)<+\infty$ for each $x\in E^{\Lambda_n}$ and $n\geqslant 1$;

2° for each $0\leqslant d<\infty$ and $n\geqslant 1$, $\{x\in E: p(x^{\Lambda_n})>d\}$ is an open set in E;

3° for each $x\in E$, $p(x^{\Lambda_n})\uparrow p(x)$ as $n\uparrow\infty$. Put $E_0=\{x\in E: p(x)<\infty\}$. The σ-algebra $\mathscr{E}_0$ is again induced by $\mathscr{E}$ on E_0.

We need the KRW-distance:

$$R_\alpha(P,\ \tilde{P})=\inf \boldsymbol{E} p_\alpha(\xi,\ \tilde{\xi})$$

on $(E^\alpha,\mathscr{E}^\alpha)$ $(\alpha\in\mathscr{S}_f)$, see [6] or [13].

Now we can state our main results:

Theorem 1. *Suppose*

i) *there exists a constant* $c\in R$ *such that*

$$\int q_n(x,\ dy)(p(y)-p(x))\leqslant c(1+p(x)),\quad x\in E_0,\ n\geqslant 1; \tag{1}$$

ii) *for each* $1\leqslant n\leqslant m$, *there exists a regular coupling* $\Omega_{n,m}$ *of* Ω_n *and* Ω_m *(see [11]) such that*

$$\begin{aligned}&\tilde{\Omega}_{n,m}p_w(x_1,\ x_2)\\ &\leqslant\sum_{u\in\Lambda_n}c_{uw}p_u(x_1,\ x_2)+c_w(n,\ m)(1+p(x_1)+p(x_2)),\end{aligned} \tag{2}$$

$$w\in\Lambda_n,\quad x_1,\ x_2\in E_0,$$

the non-diagonal elements of $(c_{uw}: u,\ w\in S)$ *and* $c_w(n,\ m)$ $(w\in\Lambda_n,\ m\geqslant n\geqslant 1)$ *being nonnegative, and*

$$\begin{aligned}&c_w(t,\ n,\ m)\\ &\equiv\sum_{k=0}^\infty\frac{t^{k+1}}{(k+1)!}[(B_n^*)^k c.(n,\ m)](w)\to 0,\ m\geqslant n\to\infty,\ t\geqslant 0,\end{aligned} \tag{3}$$

where $B_n=(c_{uv}: u,\ v\in\Lambda_n)$ *and* B_n^* *is the transpose of* B_n. *Then there exists a*

Markov process with transition probability function $P(t, x, .)$ *on state space* $(E_0, \mathscr{E}_0)$ *such that for each* $\alpha \in \mathscr{S}_f$,

$$\lim_{n\to\infty} R_\alpha(P_n(t, x, .), \ P(t, x, .)) = 0, \ x \in E_0, t \geqslant 0. \tag{4}$$

Moreover, the convergence is uniformly in $x \in E_0^{(N)} \equiv \{x \in E_0 : p(x) \leqslant N\}$. *Finally, for fixed t,* $P(t, x, .)$ *is continuous in the following sense: if* $x, \ x_n \in E_0, \ n \geqslant 1$, $\sup_n p(x_n) < \infty$ *and* $\lim_{n\to\infty} \rho_u(x_n, x) = 0$ *for every* $u \in S$, *then*

$$\lim_{n\to\infty} R_\alpha(P(t, x_n, .), \ P(t, x, .)) = 0$$

for every $\alpha \in \mathscr{S}_f$.

Clearly, if

$$\lim_{m\geqslant n\to\infty} c_u(n, m) = 0, \ u \in S; \ \sup_{m\geqslant n, u\in\Lambda_n} c_u(n, m) + \sup_u \sum_v |c_{uv}| < \infty, \tag{5}$$

then the condition (3) holds.

The condition (3) means that the interaction is decreasinvg when the distance between the components increases. The next theorem relaxes the restriction for the special p defined by

$$p(x) = \sum_{u\in S} \rho_u(x) k_u, \ x \in E. \tag{6}$$

$\mathscr{L}$ denotes the set of all Lipschitz continuous functions with respect to the above p. For $f \in \mathscr{L}$, $L(f)$ denotes the Lipschitz constant of f.

Theorem 2. *Let p be the function given by* (6). *Suppose*

i) *there exist* $c_1 \in R$ *and a nonnegative matrix* $(b(u,v): u, \ v \in S)$ *such that*

$$\begin{aligned} &\int q_n(x, dy)(\rho_v(y) - \rho_v(x)) \\ &\leqslant \beta_v + c_1\rho_v(x) + \sum_{u\in\Lambda_n} \rho_u(x) b(u, v), \\ &\qquad\qquad v \in \Lambda_n, \ x \in E_0, \ n \geqslant 1, \end{aligned} \tag{7}$$

where

$$\beta_u \geqslant 0, \ u \in S; \ \sum_u \beta_u k_u < \infty;$$

and

$$\sum_v b(u, v) k_v \leqslant M k_u, \ u \in S$$

for some $M > 0$;

ii) *for every* $1 \leqslant n \leqslant m$, *there exists a regular coupling* $\tilde{\Omega}_{n,m}$ *of* Ω_n *and* Ω_m *such that*

$$\tilde{\Omega}_{n,m} p_w(x_1, x_2) \leqslant \sum_{u\in\Lambda_n} c_{uw} p_u(x_1, x_2)$$

$$+ \sum_{u \in \Lambda_m \setminus \Lambda_n} p_u(x_2) g_{uw} + p_w(x_2) c_w(n, m), \tag{8}$$

$$w \in \Lambda_n, \ x_1, \ x_2 \in E_0,$$

where (c_{uv}), (g_{uv}), $(c_w(n, m))$ *are all nonnegative, and satisfy* (5) *and*

$$c_w(n, n) = 0, \ w \in \Lambda_n, \ n \geqslant 1; \ \sup_u \sum_v g_{uv} < \infty. \tag{9}$$

Then, there exists a Markov process with transition probability function $P(t, x, .)$ *on state space* $(E_0, \mathscr{E}_0)$ *such that*

$$R_{\Lambda_n}(P_n(t, x, .), \ P(t, x, .)) \to 0, \ n \to \infty, \ x \in E_0. \tag{10}$$

Moreover, the convergence is uniformly in finite intervals. Finally, the semigroup $\{P(t)\}_{t \geqslant 0}$ *on* $\mathscr{L}$ *defined by* $P(t, x, .)$ *has the properties*: $P(0) = I$; $P(t)$ *is a contraction in the uniform norm; there is a constant* c_3, $0 \leqslant c_3 < \infty$, *such that*

$$|P(t)f(x) - P(t)f(y)| \leqslant e^{c_3 t} L(f) p(x, y)$$

$$t \geqslant 0, \ x, y \in E_0, \ f \in \mathscr{L}. \tag{11}$$

II. Proof of Theorem 1

Let us begin with several simple lemmas in this section.

Lemma 1. *Let* $q(x) - q(x, .)$ *be a* q*-pair on a general measurable space* $(X, \mathscr{B})$, φ *and* $f(t, .)$ $(t \geqslant 0)$ *be nonnegative* $\mathscr{B}$*-measurable functions. Suppose that* $f(t, x)$ *is differentiable with respect to* t, *for every* $x \in X$, *and* $f(0, .) \geqslant \varphi$. *Then, in order to make*

$$\int P^{\min}(t, x, dy) \varphi(y) \leqslant f(t, x), \ t \geqslant 0, \ x \in X,$$

where $P^{\min}(t, x, .)$ *is the minimal* q*-process determined by the* q*-pair, it is sufficient that*

$$\frac{d}{dt} f(t, x) \geqslant \int q(x, dy)[f(t, y) - f(t, x)], \ t \geqslant 0, \ x \in X.$$

Proof. Simply use the comparison theorem [14, Theorem 6] and the backward Kolmogorov equation:

$$P(t, x, A) = \int_0^t e^{-q(x)(t-s)} \int q(x, dy) P(s, y, A) ds$$

$$+ e^{-q(x)t} \delta(x, A), \ t \geqslant 0, \ x \in X, \ A \in \mathscr{B}.$$

Without loss of generality, we will assume that the constant c in (1) is nonnegative. The next result is an immediate consequence of Lemma 1 and the condition (1).

Lemma 2. *There holds*

$$\int P_n(t, x, dy) p(y) \leqslant (1 + p(x)) e^{ct} - 1$$

$$t \geqslant 0, \ x \in E_0, \ n \geqslant 1,$$

where $P_n(t, x, .)$ *is the* q*-process determined by* $q_n(x) - q_n(x, .)$.

The next result is our main estimate.

Lemma 3. *Under the assumptions of Theorem* 1, *we have*

$$\tilde{R}^w_{n,m}(t; x_1, x_2) \equiv \int \tilde{P}_{n,m}(t; x_1, x_2; dy_1, dy_2)\, p_w(y_1, y_2)$$
$$\leqslant [e^{B_n^* t}\, p.(x_1, x_2)](w)$$
$$+ \int_0^t [(2 + p(x_1) + p(x_2))e^{cs} - 1][e^{B_n^*(t-s)} c.(n, m)](w)ds,$$
$$t \geqslant 0,\ x_1,\ x_2 \in E_0,\ w \in \Lambda_n,\ m \geqslant n \geqslant 1, \tag{12}$$

where $\tilde{P}_{n,m}(t; x_1, x_2; .)$ *is determined by* $\tilde{\Omega}_{n,m}$.

Proof. Fix $n \leqslant m$ and define column vectors:

$$R(t, x_1, x_2) = (\tilde{R}^w_{n,m}(t, x_1, x_2) : w \in \Lambda_n),$$
$$P(x_1, x_2) = (p_w(x_1, x_2) : w \in \Lambda_n),$$
$$\Phi(t, x_1, x_2) = ([(2 + p(x_1) + p(x_2))e^{ct} - 1]c_w(n, m) : w \in \Lambda_n).$$

Then, (12) is deduced as follows:

$$R(t, x_1, x_2) \leqslant e^{B_n^* t} P(x_1, x_2) + \int_0^t e^{B_n^*(t-s)} \Phi(s, x_1, x_2)ds.$$

By Lemma 2, we need only to check that .

$$e^{B_n^* t} B_n^* P(x_1, x_2) + e^{B_n^* t} B_n^* \int_0^t e^{-B_n^* s} \Phi(s, x_1, x_2)ds$$
$$+ \Phi(t, x_1, x_2)$$
$$\geqslant \int \tilde{q}_{n,m}(x_1, x_2; dy_1, dy_2) e^{B_n^* t} [P(y_1, y_2) - P(x_1, x_2)]$$
$$+ \int \tilde{q}_{n,m}(x_1, x_2; dy_1, dy_2) \int_0^t e^{B_n^*(t-s)} [\Phi(s, y_1, y_2)$$
$$- \Phi(s, x_1, x_2)]ds,$$

where $\tilde{q}_{n,m}(x_1, x_2) - \tilde{q}_{n,m}(x_1, x_2; dy_1, dy_2)$ is the q-pair determined by $\tilde{\Omega}_{n,m}$. Equivalently, the above inequality becomes

$$e^{B_n^* t} B_n^* P(x_1, x_2) + \int_0^t e^{B_n^*(t-s)} \frac{d}{ds} \Phi(s, x_1, x_2)ds + e^{B_n^* t} \Phi(0, x_1, x_2)$$
$$\geqslant \int \tilde{q}_{n,m}(x_1, x_2; dy_1, dy_2) e^{B_n^* t} [P(y_1, y_2) - P(x_1, x_2)]$$
$$+ \int \tilde{q}_{n,m}(x_1, x_2; dy_1, dy_2)[(p(y_1) - p(x_1)) + (p(y_2) - p(x_2))]$$
$$\cdot \int_0^t e^{cs} e^{B_n^*(t-s)} c.(n, m)ds.$$

But by the conditions (1), (2) and the nonnegativity of $e^{B_n^* t}$, the above right side

$$\leqslant e^{B_n^* t}\,[B_n^* P(x_1, x_2) + \Phi(0, x_1, x_2)] + \int_0^t e^{B_n^*(t-s)} \frac{d}{ds} \Phi(s, x_1, x_2) ds,$$

which completes the proof.

Now we are in a position to prove Theorem 1. Taking $x_1 = x_2 = x$, we see

$$\tilde{R}^w_{n,m}(t, x, x) \leqslant 2(1 + p(x))e^{ct} c(t, n, m),$$
$$t \geqslant 0,\ x \in E_0,\ w \in \Lambda_n,\ m \geqslant n \geqslant 1,$$

hence

$$\begin{aligned} &R_\alpha(P_n(t, x, .), P_m(t, x, .)) \\ &\leqslant \int \tilde{P}_{n,m}(t; x, x; dy_1, dy_2) p_\alpha(y_1, y_2) \\ &= \sum_{w \in \alpha} \tilde{R}^w_{n,m}(t, x, x) \\ &\leqslant 2(1 + p(x))e^{ct} \sum_{w \in \alpha} c_w(t, n, m) \to 0, \\ &\qquad m \geqslant n \to \infty. \end{aligned}$$

Starting from this fact and using [13, Theorem 2] and the proof in [6], we may construct a probability measure $P(t,x,.)$ on $(E, \mathscr{E})$ with the following properties:

a) for fixed t and $B \in \mathscr{E}_0$, $P(t, ., B)$ is $\mathscr{E}_0$-measurable;

b) for each $t \geqslant 0$ and $x \in E_0$,

$$\int P(t, x, dy) p(y) \leqslant (1 + p(x))e^{ct} - 1;$$

c) for every $\alpha \in \mathscr{S}_f$,

$$\lim_{n \to \infty} R_\alpha(P_n(t, x, .), P(t, x, .)) = 0, \text{ uniformly in } x \in F_0^{(N)}.$$

In particular, it follows that

$$P(t, x, E_0) = 1,\ t \geqslant 0,\ x \in E_0.$$

By the property c), the condition (3) and the estimate (12), we get the final assertion of Theorem 1.

To complete the proof, it remains to check the semigroup property. That is

$$\int P(t + s, x, dy) f(y) = \int P(t, x, dy) \int P(s, y, dz) f(z)$$

for each bounded $\mathscr{E}$-measurable function f. Note that if two finite measures coincide on finite dimensional open sets, then they must equal. On the other hand, each such set can be approximated by a bounded function which depends only on finite coodinates and is Lipschitz continuous with respect to p_α for some $\alpha \in \mathscr{S}_f$. Therefore, it suffices to prove the above equation for such a function f. In the following, we will fix such an f.

First, by (12), the property c) and the final assertion of the theorem, we obtain

$$\left| \int P_m(t, x^{\Lambda_m}, dy) f(y) - \int P(t, x, dy) f(y) \right|$$

$$\leqslant \left| \int P_m(t, x^{A_m}, dy) f(y) - \int P_n(t, x^{A_m}, dy) f(y) \right|$$
$$+ \left| \int P_n(t, x^{A_m}, dy) f(y) - \int P(t, x^{A_m}, dy) f(y) \right|$$
$$+ \left| \int P(t, x^{A_m}, dy) f(y) - \int P(t, x, dy) f(y) \right| \to 0,$$

for $n \to \infty$ and then $m \to \infty$. Hence

$$\lim_{m\to\infty} \int P_m(t, x^{A_m}, dy) f(y) = \int P(t, x, dy) f(y) \tag{13}$$

uniformly in x on p-bounded sets. Similarly,

$$\lim_{m\to\infty} \left[\int P_m(t, x^{A_m}, dy) f(y) - \int P(t, x^{A_m}, dy) f(y) \right] = 0 \tag{14}$$

uniformly in x on p-bounded sets.

Next, by Lemma 2, for given $\varepsilon > 0$, there is an N large enough such that

$$P_n(t, x, \{y \in E: p(y) > N\}) \leqslant \varepsilon \wedge \left(\frac{\varepsilon}{\|f\|} \right), \quad n \geqslant 1. \tag{15}$$

Finally, if we use "$A \overset{(\varepsilon)}{=} B$" to denote "$|A - B| \leqslant \varepsilon$", then we get

$$\int P(t, x, dy) \int P(s, y, dz) f(z)$$
$$= \lim_{m\to\infty} \int P(t, x, dy) \int P_m(s, y^{A_m}, dz) f(z)$$
$$= \lim_{m\to\infty} \lim_{n\to\infty} \int P_n(t, x, dy) \int_{E^{A_m}} P_m(s, y^{A_m}, dz) f(z)$$
$$\overset{(\varepsilon)}{=} \lim_{m\to\infty} \lim_{n\to\infty} \int_{E_0^{(N)}} P_n(t, x, dy) \int_{E^{A_m}} P_m(s, y^{A_m}, dz) f(z) \quad \text{(by (15))}$$
$$= \lim_{n\to\infty} \int_{E_0^{(N)}} P_n(t, x, dy) \int_{E^{A_n}} P_n(s, y^{A_n}, dz) f(z) \quad \text{(by (13))}$$
$$\overset{(\varepsilon)}{=} \lim_{n\to\infty} \int_{E_0^{(N)}} P_n(t, x, dy) \int_{E^{A_n}} P_n(s, y, dz) f(z) \quad \text{(by (14))}$$
$$\overset{(\varepsilon)}{=} \lim_{n\to\infty} \int_{E_0} P_n(t, x, dy) \int_{E_0} P_n(s, y, dz) f(z) \quad \text{(by (15))}$$
$$= \lim_{n\to\infty} \int_{E_0} P_n(t+s, x, dy) f(y)$$
$$= \int_{E_0} P(t+s, x, dy) f(y).$$

This finishes our proof.

III. Proof of Theorem 2

We first note that Lemma 2 still holds because the condition (7) implies (1). Next, using the methods in the proof of Lemma 3, we may prove the two following results:

Lemma 4. *Let (7) hold. Then there exists* $c_2 \in [0, \infty)$, *such that*

$$\int P_n(t,x,dy)\rho_v(y) \leqslant \sum_{u\in\Lambda_n}[\rho_u(x)e^{c_1t}+\beta_u e^{c_2t}] \cdot \sum_{l=0}^{\infty}\frac{t^l}{l!}b_n^{(l)}(u,v), \tag{16}$$

$$t\geqslant 0,\ x\in E_0,\ v\in\Lambda_n,\ n\geqslant 1,$$

where $(b_n^{(l)}(u,v):u,v\in\Lambda_n)$ *is the l-times product of* $(b(u,v):u,v\in\Lambda_n)$.

Lemma 5. *Under the assumptions of Theorem 2, we have*

$$\tilde{R}^w_{n,m}(t,x_1,x_2) \leqslant [e^{B_n^* t}p.(x_1,x_2)](w)$$
$$+\int_0^t ds \sum_{u\in\Lambda_m}[\rho_u(x_2)e^{c_1s}+\beta_u e^{c_2s}]\sum_{l=0}^{\infty}\frac{s^l}{l!}$$
$$\cdot[e^{B_n^*(t-s)}b^{(l)}(u,.)k.c.(n,m)](w)$$
$$+\int_0^t ds \sum_{u\in\Lambda_m}[\rho_u(x_2)e^{c_1s}+\beta_u e^{c_2s}]\sum_{l=0}^{\infty}\frac{s^l}{l!}$$
$$\cdot\sum_{v\in\Lambda_m\setminus\Lambda_n}b^{(l)}(u,v)k_v[e^{B_n^*(t-s)}g_{v;}.](w),$$
$$t\geqslant 0,\ x_1,x_2\in E_0,\ w\in\Lambda_n,\ m\geqslant n\geqslant 1.$$

By Lemma 5 and the assumptions of Theorem 2, we now obtain

$$R_{\Lambda_n}(P_n(t,x,\cdot),\ P_m(t,x,\cdot))\to 0,\ m\geqslant n\to\infty,\ t\geqslant 0,\ x\in E_0. \tag{17}$$

In particular,

$$R_a(P_n(t,x,.),\ P_m(t,x,.))\to 0,\ m\geqslant n\to\infty, \tag{18}$$
$$t\geqslant 0,\ x\in E_0,\ a\in\mathscr{S}_f.$$

From this, as we did in the last section, we can construct a probability measure on $(E_0,\mathscr{E}_0)$ with the properties a) and b) mentioned in the last section so that

$$R_{\Lambda_n}(P_n(t,x,.),\ P(t,x,.))\to 0,\ n\to\infty, \tag{19}$$
$$t\geqslant 0,\ x\in E_0.$$

However, the convergence in (19) is not necessarily uniform in $x\in E_0^{(N)}$, which is just the point why we need a different approach to prove the semigroup property.

Lemma 6. *Let* $\mathscr{G}\subset\mathscr{L}$ *satisfy* $\sup\{L(g):g\in\mathscr{G}\}<\infty$. *Then for each* $t\geqslant 0$ *and* $x\in E_0$, *we have* $(P_n(t)-P(t))g(x)\to 0$ *uniformly in* $g\in\mathscr{G}$ *as* $n\to\infty$.

Proof. Set $g_n(x)=g(x^{\Lambda_n}), x\in E_0, n\geqslant 1$, then $\sup_{n\geqslant 1}L(g_n)\leqslant L(g)$. The assertion follows from the property b), (19) and

$$|(P_n(t)-P(t))g(x)|$$
$$\leqslant|(P_n(t)-P(t))g_n(x)|+|P_n(t)(g-g_n)(x)|+|P(t)(g-g_n)(x)|$$
$$\leqslant L(g)R_{\Lambda_n}(P_n(t,x,.),\ P(t,x,.))+L(g)\sum_{u\notin\Lambda_n}p_u(x)$$
$$+L(g)\int P(t,x,dy)\sum_{u\notin\Lambda_n}p_u(y).$$

Now, by Lemma 5 and the assumptions of Theorem 2, we see that there is a constant $c_3 \in [0, \infty)$ so that

$$\begin{aligned} |P_n(t)f(x_1) - P_n(t)f(x_2)| &\leqslant L(f) \int \widetilde{P}_{n,n}(t; x_1, x_2; dy_1, dy_2) p(y_1, y_2) \\ &\leqslant L(f) \Big[\sum_{w \in \Lambda_n} \int \widetilde{P}_{n,n}(t; x_1, x_2; dy_1, dy_2) p_w(y_1, y_2) \\ &\qquad + \sum_{w \notin \Lambda_n} p_w(x_1, x_2) \Big] \\ &\leqslant e^{c_3 t} L(f) p(x_1, x_2). \end{aligned}$$

By this and Lemma 6, we get (11).

Now, it is not difficult to complete the proof of Theorem 2 by means of [5].

The author would like to thank Prof. Yan Shijiang and Huang Liping for their comments on an earlier version of this paper.

References

[1] Spitzer, F., *Adv. in Math.*, **5**(1970), 246—290.
[2] Holley, R., *Adv. in Math.*, **5**(1970), 291—309.
[3] Liggett, T. M., *Ann. of Prob.*, **1**(1973), 240—253.
[4] Andjel, E. D., *Ann. of Prob.*, **10**(1980), 527—547.
[5] Liggett, T. M. & Spitzer, F., *Z. Wahr.*, **56**(1981), 443—468.
[6] Basis, V. Ya., *Theory Probability Appl.*, **21**(1976), 727—740.
[7] Chen, Mufa, *Acta Mathematica Sinica*, New Series, **1**(1985), 3: 261—273.
[8] Zheng X. G. & Ding, W. D., Existence theorems for generalized linear growth processes, to appear in *Acta Math. Sci.*
[9] Yan Shijiang & Chen Mufa, *Chin. Ann. of Math.*, **7B**(1986), 90—110.
[10] 严士健、李占柄，物理学报，**29**(1980), 2: 139—152.
[11] Chen M. F., *Acta Mathematica Sinica*, New Series, **2**(1986), 2.
[12] Chen Mufa & Zheng Xiaogu, *Scientia Sinica*, **26**(1983), 11—24.
[13] Dobrushin, R. L., *Theory Probability Appl.*, **XV**(1970), 458—486.
[14] 陈木法，北京师大学报，**3**(1979), 66—73.
[15] Greven, A., A critical phenomenon for the "coupled branching process" (preprint).
[16] Holley, R. & Liggett, T. M., *Z. Wahr.*, **55**(1981), 165—195.
[17] 唐守正，应用概率统计，**1**(1985), 1:11—22.