Corrections of the Book "Eigenvalues, Inequalities, and Ergodic Theory" by Mu-Fa Chen

(June 12, 2009)

- page 6, line 13. $\alpha = 2^{-1}\sqrt{|K|/(d-1)}$
- page 9, line -6, in the definition of $\widetilde{W}$, add ", $w \in L^2(\pi)$ if $k = \infty$"
- page 10, line 2. $\sup_{i \geqslant 0}$
- Page 14, line 7. $(p > 1)$.
- Page 14, line 9. for some $\varepsilon > 0$. When $p = 1$, a constant $C \geqslant 1$ is added to the right-hand side.
- Page 14, line -5. long–standing
- Page 24, line -11. Remove the coefficient $\frac{1}{2}$
- Page 60, -4. Replace "Set $u_{-1} = 0$. Define" by the following:

  Set $u_{-1} = 0$. By (3.28) and the increasing property of g_i, we have

  $$\mu_n b_n u_n = -\lambda_1 \sum_{i=0}^{n} \mu_i g_i \leqslant -\lambda_1 \sum_{i \leqslant n: g_i < 0}^{n} \mu_i g_i \leqslant (-\lambda_1 g_0) \sum_{i \leqslant n: g_i < 0}^{n} \mu_i \leqslant -\lambda_1 g_0 Z < \infty.$$

  Hence $c < \infty$ and furthermore $g \in L^1(\pi)$. Define
- page 94, line 11: Landau $\longrightarrow$ E. Landau; line 12: $x^2 \mathrm{d}^2/\mathrm{d}x^2$

- Page 96, line 9. $[0, \infty)$
- page 106, 8, "$\inf_{k\geqslant k_0}$" $\longrightarrow$ "$\sup_{k\geqslant k_0}$"
- page 106, 9, Remove > 0
- page 115, in the definition of $\widetilde{\mathscr{F}}$ and $\widetilde{\mathscr{F}'}$, add ", $f \in L^2(\pi)$ if $x_0 = \infty$"
- Page 118, line -6. Remove Z
- Page 118, line -5. Remove (since $\pi(f) \geqslant 0$)
- Page 118, line -4. Replace "Thus," by the following
  Replacing f with $\bar{f}$, it follows that
- Page 119, line -3. Replace $\varphi(x \wedge \cdot)^2$ by $\varphi(x \wedge \cdot)$
- Page 128, 6. Replace the line
  "Now, let $f \in \mathscr{F}'$ satisfy $\sup_{x\in(0,D)} I\!I(f)(x) < \infty$. Take $h(x) = \int_x^D f a^{-1} e^C$." by the following.
  Now, let $f \in \mathscr{F}'$ satisfy $\sup_{x\in(0,D)} I\!I(f)(x) =: c < \infty$. Take $h(x) = \int_x^D f a^{-1} e^C$. Then we have $f I\!I(f)(x) \leqslant c f(x) < \infty$ and furthermore $H(x) = \int_x^D f I\!I(f) e^C/a \leqslant c \int_x^D f e^C/a < \infty$ for all $x \in (0, D)$.
- Page 128, 10–12. Remove the lines "When $D = \infty$, ... $M \uparrow \infty$."
- Page 128, -8. "$< \infty$" $\longrightarrow$ "$=: c < \infty$"
- Page 128, -6. "$= \sup_{x\in(0,D)} I\!I(f)(x)$" $\longrightarrow$ "$= c$"
- Page 128, -4. Replace "When $D = \infty$, there is again a problem about the integrability of g, which can be solved by using the method mentioned in the last paragraph." by
  "Here we have used the fact that $\int_x^D g e^C/a \leqslant c \int_x^D f e^C/a < \infty$."
- Page 135, line 4, $D\big(\tilde{f}\big) \leqslant D(f)$
- Page 135, line -3,-4, replace v_j by v_i in two places.
- Page 154, line -11, Chen, 2002b.
- Page 156, line 11, replace "That is the first assertion" by the following.

  Actually, we have seen that there is a $t_0 > 0$ and $\gamma \in (0, 1)$ such that $\|P_{t_0} - \pi\|_{1\to 1} \leqslant \gamma$. Given $t \geqslant 0$, express $t = mt_0 + h$ with $m \in \mathbb{N}_+$ and $h \in [0, t_0)$. Then for every f with $\pi(f) = 0$, we have $\pi(P_t f) = 0$ for all t, and furthermore

  $$\|P_t f\|_1 = \|P_{mt_0+h} f\|_1 \leqslant \|P_h f\|_1 \gamma^m \leqslant \|f\|_1 \gamma^{t/t_0 - 1} = \gamma^{-1} e^{(t_0^{-1}\log\gamma)\,t} \|f\|_1$$

  for all t. This gives the required assertion since $\log\gamma < 0$.

- Page 158, line -6, (8.2)
- Page 159, line -5, remove the word "standard".
- Page 161, 11. Replace 2000a by 2000b
- Page 178, 11, iff $\longrightarrow$ if
- Page 178, 12, first model but not the second one
- Page 182, -17. well-known
- Page 194, 10. Gronwall lemma
- Page 209, -18. **1581**: 1–114
- Page 209, -19. Springer-Verlag, 1992$\longrightarrow$Springer-Verlag, 1994
- Page 211, 2. 30A(2)
- Page 211, 14. 28(2):
- Page 211, -22. On the ergodic
- Page 211, -1. 87(2):
- Page 212, 21. preprint$\longrightarrow$ Potential Analysis, 23 (4): 303–322 (2005)
- Page 212, -16. in press$\longrightarrow$ 112–124 (2005)
- Page 212, -12. The item of M.F. Chen, L.P. Huang, and X.J. Xu is replaced by
  M.F. Chen, L.P. Huang, and X.J. Xu, Continuum limit for reaction diffusion processes with several species, in "Prob. and Stat. (Tianjin, 1988/1989)", Z.P. Jiang, S.J. Yan, P. Cheng and R. Wu (Eds.), 23–31, Nankai Ser. Pure Appl. Math. Theoret. Phys., World Sci. Publ., 1992.
- Page 212, -1. 45(4): 450–461
- Page 213, 2. 95(3)
- Page 213, 4. 23(11): 1130–1140, 37(1): 1–14,
- Page 213, 11. 349(3)
- Page 213, -9. LNM 1501, Springer, 1991
- Page 214, 6. 15(3): 407–438
- Page 214, 15. **1608**: 97–201,
- Page 215, 6. 15(3), 407–438

- Page 215, -20. **1563**: 54–88,
- Page 216, -21. preprint⟶ Ann. Appl. Prob., 15(2), 1433–1444 (2005)
- Page 218, -20. 39(4)
- Page 218, -19. preprint⟶ Stoch. Proc. Appl. 116(12): 1964–1976 (2006)
- Page 218, -14. in press⟶ 22(3): 807–812 (2006)
- Page 218, -10. in press⟶ 1022–1032
- Page 219, 10. 49(6)
- Page 220, 6. . Springer-Verlag:301–413⟶ :301–413, Springer-Verlag
- Page 221, -15. in press⟶ 22(1): 1–15 (2005)
- Page 221, -5. in press⟶ 47(5): 1001–1012
- Page 221, -4. Birth
- Page 222, 8. 14(1), 274–325,

Corrections to the preprint

- Page 69, 18. After "Refer to", add D. Bakry and M. Emery (1985),
- Page 139, 9-10. V.A. Kaimanovich
- Page 209. Add
  D. Bakry and M. Emery. Diffusions hypercontractives. *LNM*, 1123: 177–206, 1985.
- Page 223. Add Emery, M. 69
- Page 224. Kaimanovich, V.A.

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- Page 15. In the paper
  Wang, J. (2009), Criteria for functional inequalities for ergodic birth-death processes [Acta Math. Sin. 2012, 28:2, 357–370],
  it is proved that one can remove (ε) in the last line on page 15 and the last sentence in Theorem 1.10. The direct proof is also easy.

  Using the Sobolev inequality, we have shown that under the uniqueness and ergodic assumption, when $q > 2$, the Nash inequality

  $$\mathrm{Var}(f) \leqslant CD(f)^{1/p}\|f\|_1^{2/q}, \qquad f \in L^2(\pi)$$

  holds iff

  $$\sup_{n\geqslant 1} \mu[n,\infty)^{(q-2)/(q-1)} \sum_{j\leqslant n-1} \frac{1}{\mu_j b_j} < \infty.$$

  To see that the Nash inequality does not hold for $q \in (1, 2]$, rewrite it as

  $$\mathrm{Var}(f)^p \leqslant A_p D(f), \qquad f \in L^2(\pi),\ \|f\|_1 = 1, \tag{0.1}$$

  where A_p denotes the optimal constant. By the splitting technique, this is equivalent to

  $$\|f\|_2^{2p} \leqslant C_p D(f), \qquad f(0) = 0,\ f \in L^2(\pi),\ \|f\|_1 = 1,$$

  Since $\|f\|_2 \geqslant \|f\|_1 = 1$, it is clear that the last inequality becomes stronger when p increases. Thus, it is sufficient to show that the inequality (0.1) does not hold when $p = 2$ since $q \in (1, 2]$ corresponds to $p \in [2,\infty)$. As mentioned above, the inequality (0.1) holds (or equivalently, $A_p < \infty$) for $p \in (1, 2)$ iff

  $$B_p := \sup_{n\geqslant 1} \mu[n,\infty)^{2-p} \sum_{j\leqslant n-1} \frac{1}{\mu_j b_j} < \infty.$$

  Letting $p \uparrow 2$ in (0.1), because $B_2 = \infty$, it follows that $A_2 = \infty$ and so the inequality does not hold at $p = 2$.

  We mention that the restriction "$q > 2$" comes from the reduction of the Nash inequality to the Sobolev-type inequality (cf. §6.5). For the latter one, we do have a complete criterion without the restriction. Nevertheless, we have seen that the restriction "$q > 2$" is indeed sharp since the inequality does not holds for "$q \in (0, 2)$" at least for birth–death processes.

  In the above proof, we have used the fact that $\sum_n (\mu_n b_n)^{-1} = \infty$. Thus, without this assumption, for the maximal process, the inequality also holds at the end-point $p = 2$. The reason is that the corresponding Poincaré-type inequality holds for the maximal process in the region $p \in (1, 2)$ and so the proof above applies. Now, what happens if $p \in (2,\infty)$?

- Page 93, line 3. $\lambda_1 \geqslant c_1^{-1}\big(\sqrt{c_2} - \sqrt{c_2 - 1}\,\big)^2 \geqslant 1/4c_1c_2$
- Page 100. As in the discrete case, one can remove (ε) in the last line of Table 5.1 as well as the sentence below the table: "The '(ε)' in the last line ... being necessary."
- Page 110, 3–4. Need a factor 2 on the right-hand side
- Page 124, 2. $\nu/(\nu-2) \longrightarrow 2\nu/(\nu-2)$
- Page 126, 13. 2001b $\longrightarrow$ 2002a
- Page 134, -7, At the end, add "Clearly, we can assume that $f \geqslant 0$."
- Page 161, -3. $p_s(\cdot,\cdot) \in L^{1/2}(\pi)$ for some $s > 0$
- Page 161, -2. $p_s(\cdot,\cdot) \in L^{1/2}_{\text{loc}}(\pi)$ for some $s > 0$, then $\lambda_1 = \tilde{\varepsilon}_1$, where $\tilde{\varepsilon}_1$ is modified from Theorem 8.8 (2) with an addition that $C \in L^1_{\text{loc}}(\pi)$. [Remark. Clearly, $\varepsilon_1 \geqslant \tilde{\varepsilon}_1 \geqslant \varepsilon_2$. By Lemma 8.9, $\varepsilon_1 > 0$ iff $\varepsilon_2 > 0$ and so does $\tilde{\varepsilon}_1$]
- Page 162, 7. From $\longrightarrow$ By assumption, from
- Page 162, 9. $L^{1/2}_{\text{loc}}(\pi)$ by assumption $\longrightarrow$ $L^1_{\text{loc}}(\pi)$ and $\varepsilon = \tilde{\varepsilon}_1 \geqslant \lambda_1$
- Page 162, -3 $\sim$ -6. $\varepsilon_1 \longrightarrow \tilde{\varepsilon}_1$
- Page 162, -3. Combining this with Theorem 8.8 (2), $\longrightarrow$ Therefore
- Page 216, 13. birth $\longrightarrow$ Birth
- Page 154, 1. $[0,\infty) \longrightarrow [0,\infty)$

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- page 15, -4.
  Discrete spectrum $\qquad (*)\ \&\ \lim_{n\to\infty} \mu[n,\infty) \sum_{0\leqslant j\leqslant n-1} \frac{1}{\mu_j b_j} = 0$
- page 100, 10.
  Discrete spectrum $\qquad (*)\ \&\ \lim_{x\to\infty} \mu[x,\infty) \int_0^x e^{-C} = 0$
- page 183, -2. 20 $\longrightarrow$ 25
- page 184, 1. 2400 $\longrightarrow$ 2409

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- page 155, -8. (i) exponentially $\longrightarrow$ (i) π-a.s. exponentially