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Preface to the First Edition

The main purpose of the book is to introduce some progress on probability theory and its applications to physics, made by Chinese probabilists, especially by a group at Beijing Normal University in the past 15 years. Up to now, most of the work is only available for the Chinese-speaking people. In order to make the book as self-contained as possible and suitable for a wider range of readers, a fundamental part of the subject, contributed by many mathematicians from different countries, is also included. The book starts with some new contributions to the classical subject—Markov chains, then goes to the general jump processes and symmetrizable jump processes, equilibrium particle systems and non-equilibrium particle systems. Accordingly the book is divided into four parts. An elementary overlook of the book is presented in Chapter 0. Some notes on the bibliographies and open problems are collected in the last section of each chapter. It is hoped that the book could be useful for both experts and newcomers, not only for mathematicians but also for the researchers in related areas such as mathematical physics, chemistry and biology.

The present book is based on the book "Jump Processes and Particle Systems" by the author, published five years ago by the Press of Beijing Normal University. About 1/3 of the material is newly added. Even for the materials in the Chinese edition, they are either reorganized or simplified. Some of them are removed. A part of the Chinese book was used several times for graduate students, the materials in Chapter 0 was even used twice for undergraduate students in a course on Stochastic Processes. Moreover, the galley proof of the present book has been used for graduate students in their second and third semesters.

The author would like to express his warmest gratitude to Professor Z. T. Hou, Professor D. W. Stroock and Professor S. J. Yan for their teachings and advices. Their influences are contained almost everywhere in the book. In the past 15 years, the author has been benefited from a large number of colleagues, friends and students, it is too many to list individually here. However, most of their names appear in the "Notes" sections, as well as in the Bibliography and in the Index of the book. Their contributions and cooperations are greatly appreciated. The author is indebted to Professor X. F. Liu, Y. B. Li, B. M. Wang, X. L. Wang, J. Wu, S. Y. Zhang and Y. H. Zhang for reading the galley proof, correcting errors and improving the quality of the presentations. It is a nice chance to acknowledge the financial support during the past years by Fok Ying-Tung Educational Foundation, Foundation of Institution of Higher Education for Doctoral Program, Foundation of State Education Commission for Outstanding Young Teachers and the

National Natural Science Foundation of China. Thanks are also expressed to the World Scientific for their efforts on publishing the book.

M. F. Chen
Beijing
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Preface to the Second Edition

The main change of this second edition is Chapter 5 on "Probability Metrics and Coupling Methods" and Chapter 9 on "Spectral Gap" (or equivalently, "the first non-trivial eigenvalue"). Actually, these two chapters have been rewritten, within the original text. In the former chapter, the topic of "optimal Markovian couplings" is added and the "stochastic comparability" for jump processes is completed. In the latter chapter, two general results on estimating spectral gap by couplings and two dual variational formulas for spectral gap of birth-death processes are added. Moreover, a generalized Cheeger's approach is renewed for unbounded jump processes. Next, Section 4.5 on "Single Birth Processes" and Section 14.2 on "Ergodicity of Reaction-diffusion Processes" are updated. But the original technical Section 14.3 is removed. Besides, a large number of recent publications are included. Numerous modifications, improvements or corrections are made in almost every page. It is hoped that the serious effort could improve the quality of the book and bring the reader to enjoy some of the recent developments.

Roughly speaking, this book deals with two subjects: Markov Jump Processes (Parts I and Ⅱ) and Interacting Particle Systems (Parts Ⅲ and Ⅳ). If one is interested only in the second subject, it is not necessary to read all of the first nine chapters, but instead, may have a look at Chapters 4, 5, 7, 9 plus §2.3 or so. A quick way to read the book is glancing at the elementary Chapter 0, to get some impression about what studied in the book, to have some test of the results, and to choose what for the further reading. Sometimes, I feel crazy to write such a thick book, this is due to the wider range of topics. Even though it can be shorten easily by moving some details but the resulting book would be much less readable. Anyhow, I believe that the reader can make the book thin and thin.

A concrete model throughout the whole book is Schlögl's (second) model, which is introduced at the beginning (Example 0.3) to show the power of our first main result and discussed right after the last theorem (Theorem 16.3) of the book about its unsolved problems. This model, completely different from Ising model, is typical from non-equilibrium statistical physics. Its generalization is the polynomial model or more generally, the class of reaction-diffusion processes. Locally, these models are Markov chains. But even in this case, the uniqueness problem of the process was opened for several years, though everyone working in this field believes so. From physical point of view, the Markov chains should be ergodic and this is finally proved in Chapter 4. Thus, to study the phase transitions, we have to go to the infinite dimensional setting. The first hard stone is the construction of the corresponding Markov processes. For which, the mathematical tool is prepared in Chapter 5 and the construction is done in Chapter 13. The model is essentially irreversible, it can be reversible (equilibrium) only in a special case. The proof of a criterion for the reversibility is prepared in Chapter 7 and completed in Chapter 14. The topics studied in almost every chapter are either led by or related to Schlögl's model, even though sometimes it is not explicitly mentioned. Actually, the last four chapters are all devoted to the reaction-diffusion processes.

The Schlögl model possesses the main characters of the current mathematics: infinite dimensional, non-linear, complex systems and so on. It provides us a chance to re-examine the well developed finite dimensional mathematics, to create new mathematical tools or new research topics. It is not surprising that many ideas and results from different branches of mathematics, as well as physics, are used in the book. However, it is surprising that the methods developed in this book turn out to have a deep application to Riemannian geometry and spectral theory. This is clearly a different story. Since there are so much progress made in the past ten years or more, a large part of the new materials are out of the scope of this book, the author has decided to write a separate book under the title "Eigenvalues, Inequalities and Ergodic Theory".

It is a pleasure to recall the fruitful cooperation with my previous students and colleagues: Y. H. Mao, F. Y. Wang, Y. Z. Wang, S. Y. Zhang, Y. H. Zhang et al. Their contributions heighten remarkably the quality of the book.

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