Corrections of the Book "Eigenvalues, Inequalities, and Ergodic Theory" by Mu-Fa Chen

(July 21, 2025)

- page 6, line 13. $\alpha = 2^{-1}\sqrt{|K|/(d-1)}$
- page 9, line -6, in the definition of $\widetilde{W}$, add ", $w \in L^2(\pi)$ if $k = \infty$"
- page 10, line 2. $\sup_{i \geqslant 0}$
- Page 14, line 7. $(p > 1)$.
- Page 14, line 9. for some $\varepsilon > 0$. When $p = 1$, a constant $C \geqslant 1$ is added to the right-hand side.
- Page 14, line -5. long–standing
- Page 15, Discrete Spectrum $\lim_{n\to\infty} \mu[n,\infty) \sum_{j\geqslant n}$
- Page 15, 16. The extra (ε) in the last line of page 15, as well as the last sentence in Theorem 1.10 can be removed since in the case $q \in (1, 2]$, the Nash inequality does not hold, as proved in the paper: Wang, J. (2009), Criteria for functional inequalities for ergodic birth-death processes, Acta Math. Sin. 2012, 28:2, 357–370.
- Page 24, line -11. Remove the coefficient $\frac{1}{2}$
- Page 25, Example 2.6 (Synchronous coupling method)
- Page 54, -1, replace "in Corollary 1.2(1)" by the following: by Chen and Wang (1997a) to show that $\lambda_1 \geqslant \pi^2/D^2 + (1 - 2/\pi)K$, $K \geqslant 0$.

- Page 60, -4. Replace "Set $u_{-1} = 0$. Define" by the following:

  Set $u_{-1} = 0$. By (3.28) and the increasing property of g_i, we1 have

  $$\mu_n b_n u_n = -\lambda_1 \sum_{i=0}^{n} \mu_i g_i \leqslant -\lambda_1 \sum_{i\leqslant n: g_i<0}^{n} \mu_i g_i \leqslant (-\lambda_1 g_0) \sum_{i\leqslant n: g_i<0}^{n} \mu_i \\ \leqslant -\lambda_1 g_0 Z < \infty.$$

  Hence $c < \infty$ and furthermore $g \in L^1(\pi)$. Define
- page 69. line 18, D. Bakry and M. Emery (1985), Bakry (1992)→ (1994)
- page 93, line 3: $\lambda_1 \geqslant c_1^{-1}(\sqrt{c_2} - \sqrt{c_2 - 1})^2 \geqslant$
- page 94, line 12: E. Landau; $x^2\mathrm{d}^2/\mathrm{d}x^2$
- Page 96, line 9. $[0, \infty)$
- Page 100, Discrete spectrum $\lim_{x\to\infty} \mu[x, \infty) \int_0^x e^{-C} = 0$. Remove (ε) at the end of the table.
- page 106, 8, "$\inf_{k\geqslant k_0}$" $\longrightarrow$ "$\sup_{k\geqslant k_0}$"
- page 106, 9, Remove > 0
- Page 110, 3–4. Need a factor 2 on the right-hand side
- page 115, in the definition of $\widetilde{\mathscr{F}}$ and $\widetilde{\mathscr{F}'}$, add ", $f \in L^2(\pi)$ if $x_0 = \infty$"
- Page 118, line -6. Remove Z
- Page 118, line -5. Remove (since $\pi(f) \geqslant 0$)
- Page 118, line -4. Replace "Thus," by the following
  Replacing f with $\bar{f}$, it follows that
- Page 119, line -3. Replace $\varphi(x \wedge \cdot)^2$ by $\varphi(x \wedge \cdot)$
- Page 124, 2. $\nu/(\nu - 2) \longrightarrow 2\nu/(\nu - 2)$
- Page 126, 13. 2001b $\longrightarrow$ 2002a
- Page 128, 6. Replace the line
  "Now, let $f \in \mathscr{F}'$ satisfy $\sup_{x\in(0,D)} II(f)(x) < \infty$. Take $h(x) = \int_x^D f a^{-1} e^C$." by the following.
  Now, let $f \in \mathscr{F}'$ satisfy $\sup_{x\in(0,D)} II(f)(x) =: c < \infty$. Take $h(x) = \int_x^D f a^{-1} e^C$. Then we have $f II(f)(x) \leqslant c f(x) < \infty$ and furthermore $H(x) = \int_x^D f II(f) e^C/a \leqslant c \int_x^D f e^C/a < \infty$ for all $x \in (0, D)$.

- Page 128, 10–12. Remove the lines "When $D = \infty$, ... $M \uparrow \infty$." Add:
- Page 128, -8. "$< \infty$" $\longrightarrow$ "$=: c < \infty$"
- Page 128, -6. "$= \sup_{x\in(0,D)} I\!I(f)(x)$" $\longrightarrow$ "$= c$"
- Page 128, -5. Replace "When $D = \infty$, there is again a problem about the integrability of g, which can be solved by using the method mentioned in the last paragraph." by
  "Here we have used the fact that $\int_x^D g e^C/a \leqslant c \int_x^D f e^C/a < \infty$."
- Page 134, -7, At the end, add "Clearly, we can assume that $f \geqslant 0$."
- Page 135, line 4, $D\big(\tilde f\big) \leqslant D(f)$
- Page 135, line -3,-4, replace v_j by v_i in two places.
- Page 139, line 9-10, V.A. Kaimanovich
- Page 154, line -11, Chen, 2002b.
- page 155, -8. (i) exponentially $\longrightarrow$ (i) π-a.s. exponentially
- Page 156, line 11, replace "That is the first assertion" by the following.

  Actually, we have seen that there is a $t_0 > 0$ and $\gamma \in (0,1)$ such that $\|P_{t_0} - \pi\|_{1\to 1} \leqslant \gamma$. Given $t \geqslant 0$, express $t = mt_0 + h$ with $m \in \mathbb{N}_+$ and $h \in [0, t_0)$. Then for every f with $\pi(f) = 0$, we have $\pi(P_t f) = 0$ for all t, and furthermore
  $$\begin{aligned}\|P_t f\|_1 &= \|P_{mt_0+h} f\|_1 \leqslant \|P_h f\|_1 \gamma^m \leqslant \|f\|_1 \gamma^{t/t_0 - 1}\\ &= \gamma^{-1} e^{(t_0^{-1}\log\gamma)\, t} \|f\|_1\end{aligned}$$
  for all t. This gives the required assertion since $\log\gamma < 0$.
- Page 158, line -6, (8.2)
- Page 159, line -5, remove the word "standard".
- Page 161, 11. Replace 2000a by 2000b
- Page 161, -3. $p_s(\cdot,\cdot) \in L^{1/2}(\pi)$ for some $s > 0$
- Page 161, -2. $p_s(\cdot,\cdot) \in L^{1/2}_{\text{loc}}(\pi)$ for some $s > 0$, then $\lambda_1 = \tilde\varepsilon_1$, where $\tilde\varepsilon_1$ is modified from Theorem 8.8 (2) with an addition that $C \in L^1_{\text{loc}}(\pi)$.
  [Remark. Clearly, $\varepsilon_1 \geqslant \tilde\varepsilon_1 \geqslant \varepsilon_2$. By Lemma 8.9, $\varepsilon_1 > 0$ iff $\varepsilon_2 > 0$ and so does $\tilde\varepsilon_1$]
- Page 162, 7. From $\longrightarrow$ By assumption, from
- Page 162, 9. $L^{1/2}_{\text{loc}}(\pi)$ by assumption $\longrightarrow$ $L^1_{\text{loc}}(\pi)$ and $\varepsilon = \tilde\varepsilon_1 \geqslant \lambda_1$

- Page 162, -3 $\sim$ -6. $\varepsilon_1 \longrightarrow \tilde{\varepsilon}_1$
- Page 162, -3. Combining this with Theorem 8.8 (2), $\longrightarrow$ Therefore
- Page 178, 11, iff $\longrightarrow$ if
- Page 178, 12, first model but not the second one
- Page 182, -17. well-known
- page 183, -2. 20 $\longrightarrow$ 25
- page 184, 1. 2400 $\longrightarrow$ 2409
- Page 194, 10. Gronwall lemma
- Page 209, -18. **1581**: 1–114
- Page 209, -17. Springer-Verlag, 1992$\longrightarrow$Springer-Verlag, 1994
- Page 209, -15. D. Bakry and M. Emery. Diffusions and Hypercontractives. LNM 1123: 177–206, 1985.
- Page 211, 2. 30A(2)
- Page 211, 14. 28(2):
- Page 211, -22. On the ergodic
- Page 211, -1. 87(2):
- Page 212, 21. preprint$\longrightarrow$ Potential Analysis, 23 (4): 303–322 (2005)
- Page 212, -16. in press$\longrightarrow$ 112–124 (2005)
- Page 212, -12. The item of M.F. Chen, L.P. Huang, and X.J. Xu is replaced by
  M.F. Chen, L.P. Huang, and X.J. Xu, Continuum limit for reaction diffusion processes with several species, in "Prob. and Stat. (Tianjin, 1988/1989)", Z.P. Jiang, S.J. Yan, P. Cheng and R. Wu (Eds.), 23–31, Nankai Ser. Pure Appl. Math. Theoret. Phys., World Sci. Publ., 1992.
- Page 212, -1. 45(4): 450–461
- Page 213, 2. 95(3)
- Page 213, 4. 23(11): 1130–1140, 37(1): 1–14,
- Page 213, 11. 349(3)
- Page 213, -9. LNM 1501, Springer, 1991
- Page 214, 6. 23(3): 1414-1438. 1995.
- Page 214, 15. **1608**: 97–201,

- Page 215, 6. 15(3), 407–438
- Page 215, -20. **1563**: 54–88,
- Page 216, 13. birth ⟶ Birth
- Page 216, -21. preprint⟶ Ann. Appl. Prob., 15(2), 1433–1444 (2005)
- Page 218, -20. 39(4)
- Page 218, -19. preprint⟶ Stoch. Proc. Appl. 116(12): 1964–1976 (2006)
- Page 218, -14. in press⟶ 22(3): 807–812 (2006)
- Page 218, -10. in press⟶ 1022–1032
- Page 219, 10. 49(6)
- Page 220, 6. . Springer-Verlag:301–413⟶ :301–413, Springer-Verlag
- Page 221, -15. in press⟶ 22(1): 1–15 (2005)
- Page 221, -5. in press⟶ 47(5): 1001–1012
- Page 221, -4. Birth
- Page 222, 8. 14(1), 274–325,
- Page 223, 15. Emery, M. 69
- Page 224, 20. Kaimanovich, V.A. 139