Problems of Distance-Regular Graphs

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A connected simple graph $\Gamma = (X, R)$ with vertex set X, edge set R and path-length distance ∂ is said to be distance-regular if the numbers

$$p^h_{i,j} = |\{z \in X \mid \partial(x,z) = i,\ \partial(z,y) = j\}|$$

depend only on i, j and $h = \partial(x, y)$, and do not depend on the choices of vertices x and y. The Johnson graphs and the Hamming graphs are distance-regular, and many distance-regular graphs are related to classical geometries. P-polynomial association schemes, combinatorial frameworks to investigate codes and/or designs, are naturally associated to them.

In this series of seminar talks, we introduce several results in the theory of distance-regular graphs focusing on the structure theory and classification. In each lecture, we present a key problem and problems related to it.

Schedule

Nov. 21, A.M. Overview:

Definitions, basic properties, examples [1, Chapter 3], [3, Chapters 4–7, 9].

Nov. 22, A.M. Antipodal Graphs:

Graphs with $b_1 = c_{D-1}$, $k_i = k_{D-i}$, and related problems [5, 10, 11].

Nov. 22, P.M. Graphs of Order (s, t):

Absolute bound conjecture, bounds of $\ell(c, a, b)$, (s, c, a, k) bounds, circuit chasings, and strongly closed subgraphs [2, 6, 7, 8, 12, 13, 15].

Nov. 23, A.M. The Q-Polynomial Condition:

Properties derived from the Q-polynomial condition, homogeneities, characterizations [4, 9, 16].

Nov. 23, P.M. Terwillliger Algebras and their Modules:

Thin irreducible modules, tight subsets, related topics and applications [14, 17, 18].

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