北京师范大学 随机数学中心

京师数学公众报告

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题目: Growth rate of the scalar product for a supercritical branching process in a random environment and for products of positive random matrices

时间: 2023 年 12 月 21 日，周四下午 2：00-5：00
地点: 后主楼 1220

摘要: Let $Z_n = (Z_n(1), \cdots, Z_n(d))$ be a supercritical d-type branching process in an independent and identically distributed random environment $\xi = (\xi_0, \xi_1, \cdots)$, starting with $Z_0 = x$. Let M_n be the mean matrix of the offspring distribution at time n: its (i, j)-th entry $M_n(i, j)$ is the conditional expectation of the number of type j particles produced by a type i particle of generation n (so that M_n depends only on ξ_n). We establish a Kesten-Stigum type theorem for the scalar product $\langle Z_n, y\rangle$ for any non-negative vector y: we prove that under suitable conditions, $W_n^x(y) := \langle Z_n, y\rangle / \langle xM_0 \cdots M_{n-1}, y\rangle$ converges in probability to some r.v. W^x (which does not depend on y), and we give a criterion for W^x to be non-degenerate. For the proof, we introduce a martingale which has the same limit as $W_n(y)$, and we establish a Perron-Frobenius type theorem for the products of positive random matrices: we define some positives vectors $u_n, v_n > 0$ and positive scalars $\lambda_n, a_n > 0$ depending only on the environment sequence ξ, such that $(W_n^x(u_n))$ is a martingale which converges a.s. to W^x, that $M_n u_{n+1}^T = \lambda_n u_n^T$ for all n, and that uniformly in x and y, $\langle xM_0 \cdots M_n, y\rangle \sim a_n \langle u_0, x\rangle \langle v_n, y\rangle$. (Based on a joint work with Ion Grama and Thi Trang Nguyen)