北京师范大学 随机数学研究中心

学术报告

题 目: Local Approximation of Unimodular Random Networks

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摘 要: In this talk, we will state one of the most important central open problems in the graph/network limit theory, called **Aldous-Lyons Conjecture** by L. Lovász (2009), which says that any unimodular random network is a local limit of some finite network sequence. And we will explain in detail why the conjecture is greatly important whatever it is true or not; particularly why it will have far-reaching and extensive influences on probability theory, graph theory, combinatorics, group theory, operator algebra, dynamical system and ergodic theory if it holds. In conclusion, the conjecture is worth conquering by aspirants.

Local Approximation of Unimodular Random Networks

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The first example of a local limit theory for discrete structure sequences was given by the famous H. Furstenberg's corresponding principle (1977, J. Analyse Math. 31, 204-256). Roughly speaking, this principle means that every "locally convergent" sequence corresponds to a probability measure preserving system in some probability theory sense. The principle was used by Furstenberg to prove E. Szemeredi's theorem in combinatorial number theory, and played an important role in proving Green-Tao's theorem (B. Green, T. Tao. (2008). The primes contain arbitrarily long arithmetic progressions. Ann. Math. 167(2), 481-547).

A general study of discrete structural limit theories started in the early 2000's: I. Benjamini and O. Schramm (2001, EJP); L. Lovász and B. Szegedy (2006, JCTB; 2007, GAFA); C. Borgs, J. Chayes, L. Lovász, V. T. Sós, B. Szegedy and K. Vesztergombi (2006, in STOC'06); D. Aldous and R. Lyons (2007, EJP); C. Borgs, J. Chayes, L. Lovász, V. T. Sós and K. Vesztergombi (2008, Adv. Math.; 2012, Ann. Math.).

Note that there are a large number of various giant discrete structures (network/graph structures) in information and computer sciences, statistical physics, biology etc. Here a network means a graph with vertex labels and edge ones. Motivations for developing a limit theory of large discrete structures come from studying the asymptotic behaviour of giant discrete structures, the "similarity" and "proximity" among various giant discrete structures, and stochastic processes and probabilistic models (such as random walk, percolation and other statistical physics/mechanics models) on limit discrete structures, and subgraph densities in extremal combinatorics.

Surprisingly, there are two different natural sampling methods to define respectively the local limits for dense network/graph sequences and sparse ones. In the sparse situation, local limit of every finite network sequence is a unimodular random network; conversely, one of the most important central open problems in the graph/network limit theory, called **Aldous-Lyons Conjecture** by L. Lovász (2009), is stated as follows:

Conjecture. Any unimodular random network is a local limit of some finite network sequence.

This conjecture can imply every finitely generated group, and further every countable group is sofic. Recall sofic group was introduced by M. Gromov (1999, J. Euro. Math. Soc. 1(2), 109-197) when studying Gottschalk surjunctivity conjecture in symbolic dynamical system; and soficity of a finitely generated group is implied by finite approximation of Cayley diagrams (which was introduced by B. Weiss (2000, Sankhya Ser. A. 62(3), 350-359)). Here the finite approximation of Cayley diagrams means that in the framework of sparse graph limit theory, each directed Cayley graph with edges labeled by the generators is a local limit of some finite network sequence. Is every countable group sofic? Is every countable group hyperfinte? Note soficity implies hyperfiniteness. These two problems are well-known central open problems in countable group theory. Up to now, non-sofic group has not been found. The study of sofic groups is a fruitful interplay between graph limit theory and group theory.

Connes Embedding Conjecture [A. Connes. (1976). Ann. Math. 104, 73-115] is one of the most important/famous open problems in operator algebras. The importance comes from that some unexpected equivalent statements showing as this conjecture is transversal to almost all the sub-specialization of operator algebras. Notably Connes embedding conjecture is equivalent to the operator algebra version of Hilbert's 17th problem. Notice Connes embedding conjecture holds for any group von Neumann algebra if the corresponding countable group is hyperfinite. Hence Aldous-Lyons Conjecture implies that Connes embedding conjecture holds for any group von Neumann algebra.

In 2020, a negative solution to Connes embedding conjecture was obtained as a corollary of a landmark result in quantum complexity theory by Z. Ji, A. Natarajan, T. Vidick, J. Wright and H. Yuen (2020, arXiv:2001.04383). Interestingly enough, though Connes embedding conjecture is false generally, it is still unknown whether it is true for group von Neumann algebras.

Therefore, **Aldous-Lyons Conjecture** is worth conquering by aspirants. Though it holds in some special cases, there is no consensus among experts as to whether **Aldous-Lyons Conjecture** should be true or false. If **Aldous-Lyons Conjecture** holds, then it will have far-reaching and extensive influences on probability theory, graph theory, combinatorics, group theory, operator algebra, dynamical system and ergodic theory.