

# The 15th Workshop on Markov Processes and Related Topics

July 11-15, 2019

**Jilin University**

**Co-Chair:** Mu-Fa Chen (BNU), Xia Chen (JLU)

**Local Organizer:** Xia Chen, Yong Zhang

**Sponsors:** College of Mathematics, Jilin University

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**College of Mathematics, Jilin University**

**Stochastics Research Center, Beijing Normal University**

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Website: <http://math0.bnu.edu.cn/probab/Workshop2019/>

## Sign In

Grand New Century Hotel JingYue Changchun

长春华友开元名都酒店

Time: 10:00-23:00, July 10

	July 11	July 12	July 13	July 14	July 15
<b>Chairman</b>	<b>X. Chen</b>	<b>X. C. Zhang</b>	<b>F. Q. Gao</b>	<b>Z. J. Chen</b>	<b>L. M. Wu</b>
08:30-09:00	OPENING	T. S. Zhang	M. Röckner	D. Y. Chen	F. B. Xi
09:00-09:30	M.-F. Chen	Y. Q. Li	J. P. Li	H. He	J. H. Bao
09:30-10:00	J. Wang	J. L. Zhai	L. T. Yan	W. Liu	W. Sun
10:00-10:30	Tea break	Tea break	Tea break	Tea break	Tea break
<b>Chairman</b>	<b>C.-R. Hwang</b>	<b>T. S. Zhang</b>	<b>M. Röckner</b>	<b>D. Y. Chen</b>	<b>J. P. Li</b>
10:30-11:00	Z. Dong	C.-L.Wang	X. D. Li	Z. H. Li	L. M. Wu
11:00-11:30	S. Feng	Z. X. Liu	X.-Y. Wu	Z. J. Chen	X. Chen
11:30-12:00	Y. Jiao	X. M. Jiang	D. S. Wu	X. Chen	
	Lunch	Lunch	Lunch	Lunch	Lunch
<b>Chairman</b>	<b>Z. Dong</b>	<b>C.-L.Wang</b>	<b>Y. M. Xiao</b>	<b>Z. H. Li</b>	
14:30-15:00	A. H. Xia	C. G. Yuan	Z. W. Cheng	Y. Y. Liu	
15:00-15:30	Y. H. Mao	J. H. Shao	R. J. Fang	H. Z. Zhao	
15:30-16:00	C. S. Deng	G. Q. Lan	H. Y. Li	M. Choi	
16:00-16:30	Tea break	Tea break	Y. S. Li	Tea break	
<b>Chairman</b>	<b>A. H. Xia</b>	<b>C. G. Yuan</b>	<b>Y. Q. Zhao</b>	<b>S. Feng</b>	
16:30-17:00	Y. Q. Zhao	F. Q. Gao	M. Y. Wu	X. C. Zhang	
17:00-17:30	N. S. Gao	Y. M. Xiao	S. X. Zhang	P. Kim	
17:30-18:00	X. Yang	S. Liang	X. Y. Zhang		

## July 11

**Chairman: Xia Chen**

08:30-09:00 *Opening & Taking Photos*

09:00-09:30 Mu-Fa Chen (Beijing Normal University, Beijing)

*Reconstructing the Sanxian's Music Score by a Birth-Death Matrix*

09:30-10:00 Jian Wang (Fujian Normal University, Fuzhou)

*Heat kernel estimates for symmetric pure jump Dirichlet forms*

10:00-10:30 Tea break

**Chairman: Chii-Ruey Hwang**

10:30-11:00 Zhao Dong (Chinese Academy of Sciences, Beijing)

*Large deviation principles for first-order scalar conservation laws with stochastic forcing*

11:00-11:30 Shui Feng (McMaster University, CA)

*Bayesian nonparametric analysis of kingman's coalescent*

11:30-12:00 Yong Jiao (Central South University, Changsha)

*Recent advances on the theory of noncommutative differential subordinate martingale*

**Chairman: Zhao Dong**

14:30-15:00 Aihua Xia (University of Melbourne, Australia)

*On moderate deviations in poisson approximation*

15:00-15:30 Yonghua Mao (Beijing Normal University, Beijing)

*The hitting times for Markov chains*

15:30-16:00 Changsong Deng (Wuhan University, Wuhan)

*Asymptotic formulas for the heat kernels of space and time fractional equations*

16:00-16:30 Tea break

**Chairman: Aihua Xia**

16:30-17:00 Yiqiang Zhao (Carleton University, CA)

*$\varepsilon$ -Nash equilibrium for jsq models*

17:00-17:30 Niushan Gao (Ryerson University, CA)

*On switching probability measures*

17:30-18:00 Xu Yang (North Minzu University, Yinchuan )

*On the extinction-extinguishing for a stochastic lotka-volterra type population dynamical system*

## July 12

**Chairman: Xicheng Zhang**

08:30-09:00 Tusheng Zhang (University of Manchester, UK; University of Science and Technology of China, Hefei)

*Quadratic transportation cost inequalities under uniform distance for stochastic reaction diffusion equations driven by multiplicative space-time white noise*

09:00-09:30 Yuqiang Li (East China Normal University, Shanghai)

*Deviations on record numbers of simple random walks*

09:30-10:00 Jianliang Zhai (University of Science and Technology of China, Hefei)

*Well-posedness and large deviations for 2-D Stochastic Navier-Stokes equations driven by multiplicative Lévy noise*

10:00-10:30 Tea break

**Chairman: Tusheng Zhang**

10:30-11:00 Chia-Li Wang (National Dong Hwa University, Hualien)

*Queueing Networks with Interchangeable, Cooperative Servers*

11:00-11:30 Zhenxin Liu (Dalian University of Technology, Dalian)

*The limit distribution of inhomogeneous Markov processes and Kolmogorov's problem*

11:30-12:00 Xiaomeng Jiang (Jilin University, Changchun)

*Affine periodic solutions in distribution of stochastic differential equations*

**Chairman: Chia-Li Wang**

14:30-15:00 Chenggui Yuan (Swansea University, UK)

*Stability of regime-switching processes under perturbation of transition rate matrices*

15:00-15:30 Jinghai Shao (Tianjin University, Tianjin)

*The existence of optimal control for continuous-time Markov decision processes in random environments*

15:30-16:00 Guangqiang Lan (Beijing University of Chemical Technology)

*Exponential stability of exact solution and Stochastic theta method for a kind of stochastic Volterra integro-differential equations*

16:00-16:30 Tea break

**Chairman: Chenggui Yuan**

16:30-17:00 Fuqing Gao (Wuhan University, Wuhan)

*Moderate deviations for linear statistics of  $\beta$ -ensembles*

17:00-17:30 Yimin Xiao (Michigan State University, USA)

*Fractal Dimension Results for Lévy and Lévy-Type Processes*

17:30-18:00 Song Liang (Waseda University, Japan)

*A mechanical model of brownian motion including low energy light particles*

## July 13

**Chairman: Fuqing Gao**

08:30-09:00 Michael Röckner (Bielefeld University, DE)

*The evolution to equilibrium of solutions to nonlinear Fokker–Planck equations*

09:00-09:30 Junping Li (Central South University, Changsha)

*Large deviation rates for branching processes*

09:30-10:00 Litan Yan (Donghua University, Shanghai)

*Principal values of some integral functionals of fractional Brownian motion*

10:00-10:30 Tea break

**Chairman: Michael Röckner**

10:30-11:00 Xiangdong Li (Chinese Academy of Sciences, Beijing)

*On Shannon entropy power Inequality on Riemannian manifolds and super Ricci flows*

11:00-11:30 Xian-Yuan Wu (Capital Normal University, Beijing)

*On a law of large number for last passage percolation on complete graph*

11:30-12:00 Dongsheng Wu (University of Alabama-huntsville, USA)

*Weak convergence of martingales and its application to nonlinear cointegrating regression model*

**Chairman: Yimin Xiao**

14:30-14:50 Zhiwen Cheng (Beijing Normal University, Beijing)

*Variational principles for non-symmetric markov chains*

14:50-15:10 Rongjuan Fang (Beijing Normal University, Beijing)

*Continuous-state branching processes in temporally varying environments*



15:10-15:30 Heyu Li (Jilin University, Changchun)

*Precise moment asymptotics for the stochastic parabolic Anderson model of a time-derivative Gaussian noise*

15:30-15:50 Yueshuang Li (Beijing Normal University, Beijing)

*Improved global algorithms for maximal eigenpair*

15:50-16:30 Tea break

**Chairman: Yiqiang Zhao**

16:30-16:50 Mingyan Wu (Wuhan University, Wuhan)

*Schauder's estimate for nonlocal equations with singular Lévy measures*

16:50-17:10 Shuxiong Zhang (Beijing Normal University, Beijing)

*Moderate deviation probabilities for empirical distribution of the branching random walk*

17:10-17:30 Xiaoyue Zhang (Beijing Normal University, Beijing)

*Asymptotic behaviour of heavy-tailed branching processes in random environments*

## July 14

**Chairman: Zengjing Chen**

08:30-09:00 Dayue Chen (Peking University, Beijing)

*Limit theorems for the tagged particle in exclusion processes on regular trees*

09:00-09:30 Hui He (Beijing Normal University, Beijing)

*Lower deviation and moderate deviation probabilities for maximum of a branching random walk*

09:30-10:00 Wei Liu (Wuhan University, Wuhan)

*Long-time behaviors of mean-field interacting particle systems related to McKean-Vlasov equation*

10:00-10:30 Tea break

**Chairman: Dayue Chen**

10:30-11:00 Zenghu Li (Beijing Normal University, Beijing)

*Ergodicities and exponential ergodicities of branching processes with immigration*

11:00-11:30 Zengjing Chen (Shandong University, Jinan)

*Central Limit Theorems for Sets of Probability Measures*

11:30-12:00 Xin Chen (Shanghai Jiao Tong University, Shanghai)

*Homogenization of symmetric stable-like processes in ergodic media*

**Chairman: Zenghu Li**

14:30-15:00 Yuanyuan Liu (Central South University, Changsha)

*On geometric and algebraic transience for block-structured Markov chains*

15:00-15:30 Huaizhong Zhao (Loughborough University, UK)

*Existence of geometric ergodic periodic measures of stochastic differential equations*

15:30-16:00 Michael Choi (The Chinese University of Hong Kong, Shenzhen)

*Accelerated simulated annealing with fast cooling*

16:00-16:30 Tea break

**Chairman: Shui Feng**

16:30-17:00 Xicheng Zhang (Wuhan University, Wuhan)

*Stochastic lagrangian path for leray solutions of 3-d navier-stokes equations*

17:00-17:30 Panki Kim (Seoul National University, Korea)

*Heat kernel estimates for symmetric jump processes with general mixed polynomial growths*

## July 15

**Chairman: Liming Wu**

08:30-09:00 Fubao Xi (Beijing Institute of Technology, Beijing)

*Regime-switching jump diffusions with non-lipschitz coefficients and countably many switching states*

09:00-09:30 Jianhai Bao (Central South University, Changsha)

*Convergence rate of EM algorithm for SDEs under integrability condition*

09:30-10:00 Wei Sun (Concordia University, CA)

*The three-dimensional gaussian product inequality*

10:00-10:30 Tea break

**Chairman: Junping Li**

10:30-11:00 Liming Wu (Chinese Academy of Sciences, Beijing)

*Transportation-information inequality for distance time markov chains*

11:00-11:30 Xia Chen (Jilin University, Changchun; University of Tennessee, USA)

*Parabolic Anderson models — large scale asymptotics*

## CONVERGENCE RATE OF EM ALGORITHM FOR SDES UNDER INTEGRABILITY CONDITION

**Jianhai BAO** *Central South University, China*, E-mail: jianhaibao@csu.edu.cn

**Abstract:** Numerical approximations of SDEs with continuous coefficients (e.g., global Lipschitz, non-globally Lipschitz, Holder continuous coefficients) have been investigated considerably. Nowadays numerical analysis of SDE with discontinuous coefficients also receives much attention. Whereas, most of the existing literatures focus on the setup that the drift terms obey piecewise Lipschitz conditions. In this talk we aim to investigate convergence rate of EM scheme for SDEs with integrability drifts, which allows the drifts to be much more singular. Our methods are based on heat kernel estimate and Zvonkin's transform.

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## LIMIT THEOREMS FOR THE TAGGED PARTICLE IN EXCLUSION PROCESSES ON REGULAR TREES

**Dayue CHEN** *Peking University, China*, E-mail: dayue@math.pku.edu.cn

**Abstract:** We consider exclusion processes on a rooted  $d$ -regular tree. We start from a Bernoulli product measure conditioned on having a particle at the root, which we call the tagged particle. For  $d > 2$ , we show that the tagged particle has positive linear speed and satisfies a central limit theorem. We give an explicit formula for the speed. As a key step in the proof, we first show that the exclusion process "seen from the tagged particle" has an ergodic invariant measure. The talk is based on a joint paper with Peng Chen, Nina Gantert, Dominik Schmid.

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## RECONSTRUCTING THE SANXIAN'S MUSIC SCORE BY A BIRTH-DEATH MATRIX

**Mu-Fa CHEN** *Beijing Normal University, China*, E-mail: mfchen@bjtu.edu.cn

**Abstract:** The Sanxian is a traditional Chinese three-stringed plucked instrument. Its music can be generated by tridiagonal complex matrices. The sound people hear is determined by its spectrum and naturally requires that the matrix has a real spectrum. As in quantum mechanics, the description of the model is a complex operator and the observable measurement is real. In other words, the tridiagonal complex matrix described is a self-adjoint operator on the complex inner product space with respect to a measure. It is well known that the birth-death  $Q$  matrix can be matched and is naturally self-adjoint. We will introduce the latest representative results: for a fairly wide range of self-adjoint tridiagonal complex matrices, a birth-death  $Q$  matrix can always be constructed to make both isospectral (in simple words, both have the same eigenvalues). This problem is simple and easy to understand. But we have studied it from three different perspectives: probability theory, statistical physics and computational mathematics during different periods, and have gone through a long time of exploration. We may also mention some application of this key result to Quantum Mechanics.

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## PARABOLIC ANDERSON MODELS —LARGE SCALE ASYMPTOTICS

**Xia CHEN** *Jilin University, Changchun; University of Tennessee, USA*, E-mail: xchen@math.utk.edu

**Abstract:** The model of the parabolic Anderson equation

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + W^H u$$

is relevant to some problems arising from physics such as the particle movement in disordered media, population dynamics, and the KPZ equations through a suitable transform.

In this talk we provide a survey on the progress achieved in the recent years in the large scale asymptotics for the solution. The topics include intermittency, high moment asymptotics, quenched time and space asymptotics.

## HOMOGENIZATION OF SYMMETRIC STABLE-LIKE PROCESSES IN ERGODIC MEDIA

**Xin CHEN** *Shanghai Jiaotong University, China*, E-mail: chenxin217@sjtu.edu.cn

**Abstract:** We will prove the homogenization for the stable-like process in ergodic random media which is associated with some symmetric Dirichlet form with quite general expression. In particular, the coefficients in the jumping kernel of Dirichlet forms and density function for the reference measure are allowed to be degenerate. This talk is based on a joint work with Zhen-qing Chen, Takashi Kumagai and Jian Wang.

## CENTRAL LIMIT THEOREMS FOR SETS OF PROBABILITY MEASURES

**Zengjing CHEN** *Shandong University, China*, E-mail: zjchen@sdu.edu.cn

**Abstract:** In this paper, we present two Central Limit Theorems *CLTs* for models where random events are describable by nonsingleton sets of probability measures. Such sets arise in economics and finance as the subjective prior beliefs of an agent within a model who does not have sufficient information to justify reliance on a single probability measure, in mathematical statistics and econometrics where they represent the predictions of the theory being tested or estimated empirically and where predictions are multivalued because the theory is incomplete and as representations of indeterminate objective realities. We refer to such situations as featuring "ambiguity."

Joint work with Larry G. Epstein

## VARIATIONAL PRINCIPLES FOR NON-SYMMETRIC MARKOV CHAINS

**Zhi-Wen CHENG** *Beijing Normal University, China*, E-mail: 201731130029@mail.bnu.edu.cn  
**Yong-Hua MAO** *Beijing Normal University, China*, E-mail: maoyh@bnu.edu.cn

**Abstract:** Non-symmetric Markov chains are difficult to deal with than the symmetric ones. Inspired by Dolye(1994), and Gaudillièrè and Landim(2014), we give the variational principles of the capacity of a set and the additive functional. These results generalize the variational principles of the classical capacity for symmetric Markov chains(Kemeny, Snell and Knapp(1976)) and the hitting times in Huang and Mao(2018).

## ACCELERATED SIMULATED ANNEALING WITH FAST COOLING

**Michael CHOI** *The Chinese University of Hong Kong, Shenzhen, China*, E-mail: michaelchoi@cuhk.edu.cn

**Abstract:** Originated from statistical physics, simulated annealing is a popular stochastic optimization algorithm that has found extensive empirical success in disciplines ranging from image processing to statistics and combinatorial optimization. At the heart of the algorithm lies in constructing a non-homogeneous Markov process that converges to the set of global minima as the temperature cools down. In this talk, we will first review the classical theory for simulated annealing and discuss some of its theoretical limitations. We will then introduce a promising accelerated variant of simulated annealing that provably converges faster and does not suffer from the drawbacks of its classical counterpart. This talk is based on <http://arxiv.org/abs/1901.10269>.

## ASYMPTOTIC FORMULAS FOR THE HEAT KERNELS OF SPACE AND TIME FRACTIONAL EQUATIONS

**Changsong DENG** *Wuhan University, China*, E-mail: dengcs@whu.edu.cn

**Abstract:** I will talk about the asymptotic behaviour of the fundamental solutions (heat kernels) of non-local equations with fractional operators in time and space. In particular, we establish exact asymptotic formulas for the fundamental solutions to the  $n$ -dimensional fractional heat equations in both time and space.

## LARGE DEVIATION PRINCIPLES FOR FIRST-ORDER SCALAR CONSERVATION LAWS WITH STOCHASTIC FORCING

**Zhao DONG** *Chinese Academy of Sciences, China*, E-mail: dzhao@amt.ac.cn

**Abstract:** In this paper, we established the Freidlin-Wentzell type large deviation principles for first-order scalar conservation laws perturbed by small multiplicative noise. Due to the lack of the viscous terms in the stochastic equations, the kinetic solution to the Cauchy problem for

these first-order conservation laws is studied. Then, based on the well-posedness of the kinetic solutions, we show that the large deviations holds by utilising the weak convergence approach. This is joint work with Wu Jiang Lun, Zhang Rang Rang, Zhang Tu sheng.

## CONTINUOUS-STATE BRANCHING PROCESSES IN TEMPORALLY VARYING ENVIRONMENTS

Rongjuan FANG *Beijing Normal University, China*, E-mail: fangrj@mail.bnu.edu.cn

**Abstract:** A temporally inhomogeneous continuous-state process (CB-process) is the probabilistic model for the evolution of a large population of small individuals in a varying environment. A construction of such process was given by Bansaye and Simatos (*Electron. J. Probab.* 2015) as the rescaling limit of a sequence of discrete models. The conditions for existence of the process given by Bansaye and Simatos depend not only on the ingredients in the characterization of its transition semigroup but also on its discrete approximating sequence. In this work, we provide a direct construction of the transition semigroup of the inhomogeneous CB-process under natural conditions independent of the approximating sequence. This existence result improves considerably that of Bansaye and Simatos. We also establish some stochastic integral equations of the process in the light of Dawson and Li (*Ann. Probab.* 2006/2012), which unfold interesting structures of its trajectories. This is a joint work with Zenghu Li.

## BAYESIAN NONPARAMETRIC ANALYSIS OF KINGMAN'S COALESCENT

Shui FENG *McMaster University, Canada*, E-mail: shuifeng@mcmaster.ca

KEY WORDS: Bayesian nonparametrics, Dirichlet process, Fleming-Viot process, Infinitely-many-neutral-alleles model, Kingman coalescent...

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 62C10; secondary 62M05

**Abstract:** Kingman's coalescent describes the genealogy of a population whose genetic composition evolves in time according to the class of Fleming-Viot processes with parent independent mutation ([1]). Ancestral inference can be done through Kingman coalescent based on random samples taking at each fixed time. The focus of this talk is on the Bayesian nonparametric predictive approach to ancestral inference and the interplay between the Bayesian nonparametric analysis and Kingman coalescent. The talk is based on a joint work with Stefano Favaro and Paul Jenkins ([2]).

### References

- [1] S. Ethier & R.C. Griffiths (1993). The transition function of a Fleming-Viot process, *Ann. Probab.*, **21**, 1571-1590.
- [2] S. Favaro, S. Feng & P. Jenkins (2019). Bayesian nonparametric analysis of Kingman's coalescent. *Annales de l'Institut Henri Poincaré*, Vol. **55**, No. 2, 1087-1115.



## MODERATE DEVIATIONS FOR LINEAR EIGENVALUE STATISTICS OF $\beta$ -ENSEMBLES

**Fuqing GAO** *School of Mathematics and Statistics, Wuhan University, China*, E-mail: fkgao@whu.edu.cn

**Jianyong Mu** *School of Mathematics and Statistics, Wuhan University, China*, E-mail: jianyongmu@whu.edu.cn

**Abstract:** We establish a moderate deviation principle for linear eigenvalue statistics of  $\beta$ -ensembles. The main ingredient is to obtain uniform estimates for the correlators of a family of perturbations of linear eigenvalue statistics of  $\beta$ -ensembles using the loop equations.

## ON SWITCHING PROBABILITY MEASURES

**Niushan GAO** *Ryerson University, CA*, E-mail: niushan@ryerson.ca

**Abstract:** The Fundamental Theorem of Asset Pricing is a prominent example where switching underlying probability measures plays a crucial and essential role. It is henceforth of great interest to study how certain analytical and probabilistic properties of a set behave when the underlying probability measure is switched from one to another.

Let  $\mathcal{K}$  be a convex bounded set in  $\mathbb{L}^0(\mathbb{P})$ . We study the following question. When does there exist  $\mathbb{Q} \sim \mathbb{P}$  such that  $\mathcal{K}$  is  $\mathbb{Q}$ -uniformly integrable? A closely related question asks when there exists  $\mathbb{Q} \sim \mathbb{P}$  such that a sequence in  $\mathcal{K}$  converging in probability also converges in the norm of  $\mathbb{L}^1(\mathbb{Q})$ ?

This talk is based on joint work with Denny H. Leung (National University of Singapore) and Foivos Xanthos (Ryerson University).

## LOWER DEVIATION AND MODERATE DEVIATION PROBABILITIES FOR MAXIMUM OF A BRANCHING RANDOM WALK

**Hui HE** *Beijing Normal University, China*, E-mail: hehui@bnu.edu.cn

**Abstract:** Given a super-critical branching random walk on real line started from the origin, let  $M_n$  be the maximal position of individuals at the  $n$ -th generation. Under some mild conditions, it is known from Aïdékon (2013) that as  $n \rightarrow \infty$ ,  $M_n - x^*n + \frac{3}{2\theta^*} \log n$  converges in law for some suitable constants  $x^*$  and  $\theta^*$ . In this work, we investigate its moderate deviation, in other words, the convergence rates of

$$P\left(M_n \leq x^*n - \frac{3}{2\theta^*} \log n - \ell_n\right),$$

for any positive sequence  $(\ell_n)$  such that  $\ell_n = O(n)$  and  $\ell_n \uparrow \infty$ . As a by-product, we obtain lower deviation of  $M_n$ ; i.e., the convergence rate of

$$P(M_n \leq xn),$$

for  $x < x^*$  in Böttcher case where the offspring number is at least two. We also apply our techniques to study the small ball probability of the limit of the so-called derivative martingale.

Our results complete those in Hu (2016) and Gantert and Höfelsauer (2018). This talk is based on a joint work with Xinxin Chen.

## AFFINE PERIODIC SOLUTIONS IN DISTRIBUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS

Xiaomeng JIANG *Jilin University, China*, E-mail: [jxmlucy@sina.com](mailto:jxmlucy@sina.com)

**Abstract:** It is a natural question whether periodic systems admit periodic solutions. For stochastic affine periodic systems, we demonstrate a law of large numbers including Halanay-type criterion and a LaSalle-type stationary oscillation principle to obtain the existence and stability of affine periodic solutions in distribution. As applications, we present the existence and asymptotic stability of stochastic affine periodic solutions in distribution via Lyapunov's method.

## RECENT ADVANCES ON THE THEORY OF NONCOMMUTATIVE DIFFERENTIALLY SUBORDINATE MARTINGALES

Yong JIAO *Central South University, China*, E-mail: [jiaoyong@csu.edu.cn](mailto:jiaoyong@csu.edu.cn)

**Abstract:** In this talk we will present some new advances regarding the theory of noncommutative differentially subordinate martingales. The classical differential subordination of martingales, introduced by Burkholder in the eighties, is generalized to the noncommutative setting. Working under this domination, we establish the strong-type inequalities with the constants of optimal order as  $p \rightarrow 1$  and  $p \rightarrow \infty$ , and the corresponding endpoint weak-type (1,1) estimate. In contrast to the classical case, we need to introduce two different versions of noncommutative differential subordination, depending on the range of the exponents. For the  $L^p$ -estimate,  $2 \leq p < \infty$ , a certain weaker version is sufficient; on the other hand, the strong-type  $(p, p)$  inequality,  $1 < p < 2$ , and the weak-type (1,1) estimate require a stronger version. We also introduce a notion of strong differential subordination of noncommutative semimartingales, establish the maximal weak-type (1,1) inequality under the additional assumption that the dominating process is a submartingale, and show the corresponding strong-type  $(p, p)$  estimate for  $1 < p < \infty$  under the assumption that the dominating process is a nonnegative submartingale. Finally, we give some estimates of square functions for noncommutative differentially subordinate martingales. This is accomplished by combining several techniques, including interpolation flavor method, Doob-Meyer decomposition, a significant extension of the maximal weak-type estimate of Cuculescu and a Gundy-type decomposition of an arbitrary noncommutative submartingale. This is several joint work with Adam Osekowski, Lian Wu, Narcisse R. and Dejian Zhou.

## HEAT KERNEL ESTIMATES FOR SYMMETRIC JUMP PROCESSES WITH GENERAL MIXED POLYNOMIAL GROWTHS

Panki KIM *Seoul National University, South Korea*, E-mail: [pkim@snu.ac.kr](mailto:pkim@snu.ac.kr)

**Abstract:** In this talk, we consider a symmetric pure jump Markov process  $X$  on a general metric measure space that satisfies volume doubling conditions. We discuss estimates of the transition density  $p(t, x, y)$  of  $X$  and their stabilities when the jumping kernel for  $X$  has general mixed polynomial growths. The rate function which gives growth of jumps of  $X$  may not be comparable to the scale function which provides the borderline for  $p(t, x, y)$  to have either near-diagonal estimates or off-diagonal estimates. Under the assumption that the lower scaling index of scale function is strictly bigger than 1, we establish stabilities of heat kernel estimates. If underlying metric measure space admits a conservative diffusion process which has a transition density satisfying a general subGaussian bounds, we obtain heat kernel estimates. In this case, scale function is explicitly given by the rate function and the function  $F$  related to walk dimension of underlying space. As an application, we have that the finite moment condition in terms of  $F$  on such symmetric Markov process is equivalent to a generalized version of Khintchine-type law of iterated logarithm at the infinity. This talk is based on joint works with Joohak Bae, Jaehoon Kang and Jaehun Lee.

## EXPONENTIAL STABILITY OF EXACT SOLUTION AND STOCHASTIC $\theta$ -METHOD FOR A CLASS OF STOCHASTIC VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

**Guangqiang LAN** *Beijing University of Chemical Technology, China*, E-mail: langq@mail.buct.edu.cn

**Abstract:** In [P. Hu and C. Huang, The stochastic  $\theta$ -method for nonlinear stochastic Volterra integro-differential equations, Abstract and Applied Analysis, Volume 2014, Article ID 583930, 13 pages], the authors investigated mean square convergence and mean square asymptotical stability of stochastic  $\theta$ -method for a class of nonlinear stochastic Volterra integro-differential equations. They presented sufficient conditions of the mean square asymptotical stability. However the convergence rate of the stochastic  $\theta$ -method to 0 is not known. We obtain the mean square exponential stability of the corresponding numerical method under weaker sufficient conditions than Hu and Huang's. Roughly speaking, we obtain a stronger conclusion under weaker conditions.

## PRECISE MOMENT ASYMPTOTICS FOR THE STOCHASTIC PARABOLIC ANDERSON MODEL OF A TIME-DERIVATIVE GAUSSIAN NOISE

**Heyu LI** *Jilin University, China*, E-mail: heyul15@mails.jlu.edu.cn

**Abstract:** In this talk, we investigate the precise asymptotics of

$$\mathbb{E}u^m(t, x) \quad (t \rightarrow \infty \quad \text{or} \quad m \rightarrow \infty)$$

for the stochastic parabolic Anderson model

$$\frac{\partial u}{\partial t}(t, x) = \frac{1}{2}\Delta u(t, x) + u(t, x)\frac{\partial W}{\partial t}(t, x)$$

with the time-derivative Gaussian noise  $\frac{\partial W}{\partial t}(t, x)$  that is fractional in time and homogeneous or asymptotic homogeneous in space. Our study is motivated partially by the intermittency problem for the parabolic Anderson models. (Joint work with Prof. Xia Chen)

# LARGE DEVIATION RATES FOR BRANCHING PROCESSES

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KEY WORDS: Large deviation; supercritical branching process; branching  $Q$ -matrix.

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60J27, Secondary 60J35

**Abstract:** Let  $\{X(t); t \geq 0\}$  be the continuous-time branching process with offspring rates  $\{b_k; k \geq 0\}$ . We assume that  $b_0 = 0$  and  $m = \sum_{k=1}^{\infty} kb_k < \infty$ . Let  $W(t) = e^{-mt}X(t)$  and  $W = \lim_{t \rightarrow \infty} W(t)$ . In this paper, we consider the rates of convergence to 0 as  $t \rightarrow \infty$  of

$$P\left(\left|\frac{X(t+s)}{X(t)} - e^{ms}\right| > \varepsilon\right), \quad P(|W(t) - W| > \varepsilon),$$

and

$$P\left(\left|\frac{X(t+s)}{X(t)} - e^{ms}\right| > \varepsilon | W \geq \alpha\right)$$

for  $\varepsilon > 0$  and  $\alpha > 0$  under various moment conditions on  $\{b_k; k \geq 0\}$ . It is shown that the rate of the first one is geometric, while the other two are supergeometric.

## References

- [1] ANDERSON, W. (1991). *Continuous-Time Markov Chains: An Applications-Oriented Approach*. Springer-Verlag, New York.
- [2] ATHREYA, K.B. (1994). Large Deviation Rates for Branching Processes–I. Single Type Case. *The Annals of Applied Probability*, 4(3):779-790.
- [3] ATHREYA, K.B. AND NEY, P.E. (1972). *Branching Processes*. Springer, Berlin.
- [4] CHEN, A.Y., LI, J.P. AND RAMESH, N.I. (2005). Uniqueness and Extinction of Weighted Markov Branching Processes. *Methodology and Computing in Applied Probability*, 7, 489-516.
- [5] CHOW, Y.S. AND TEICHER, H. (1988). *Probability Theory Probability Theory: Independence, Interchangeability, Martingales*. Springer, New York.
- [6] HARRIS, T.E. (1963). *The theory of branching processes*. Springer, Berlin and New York.
- [7] KARP, R. AND ZHANG, Y. (1983). Tail probabilities for finite supercritical branching processes. *Technical Report, Dept. Computer Science and Engineering, Southern Methodist University, Dallas TX*.
- [8] MILLER, H.I. AND O'SULLIVAN, J.A. (1992). Entropies and combinatorics of random branching processes and context free languages. *IEEE Trans. Inform. Theory*, 38, 1292-1311.
- [9] LI, L.Y. AND LI, J.P. (2018). Large deviation rates for supercritical branching processes with immigration. , manuscript.
- [10] LIU, J.N. AND ZHANG, M. (2016). Large deviation for supercritical branching processes with immigration. *Acta Mathematica Sinica, English Series*, 32(8):893-900.

- [11] SUN, Q. AND ZHANG, M. (2017). Harmonic moments and large deviations for supercritical branching processes with immigration. *Frontiers of Mathematics in China*, 12(5):1201-1220.

## ON SHANNON ENTROPY POWER INEQUALITY ON RIEMANNIAN MANIFOLDS AND SUPER RICCI FLOWS

Xiang-Dong LI *AMSS, Chinese Academy of Sciences*, E-mail: xdli@amt.ac.cn

**Abstract:** In his fundamental paper *The Mathematical Theory of Communication*, Claude Shannon introduced the notion of differential entropy and proved the Entropy Power Inequality (EPI) on Euclidean space. In this talk, we present our recent work on the extension of EPI on Riemannian manifolds and on super Ricci flows. Joint work with Songzi Li.

## IMPROVED GLOBAL ALGORITHMS FOR MAXIMAL EIGENPAIR

Yue-Shuang LI *Beijing Normal University, China*, E-mail: liyueshuang@bnu.edu.cn

**Abstract:** Based on our previous global algorithms for computing the maximal eigenpair introduced in a rather general setup. The efficiency of the global algorithms is improved in terms of a good use of the power iteration and some quasi-symmetric techniques. A typical example is introduced to illustrate these techniques.

## DEVIATIONS ON RECORD NUMBERS OF SIMPLE RANDOM WALKS

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KEY WORDS: Number of Ladder points, moderated deviation, small ball probability.

MATHEMATICAL SUBJECT CLASSIFICATION: 60F10, 60F17, 60G50, 60K05

**Abstract:** Simple random walk is a fundamental but useful stochastic model. In this talk, some new results related to the number of the ladder points of simple random walks will be introduced, including the limit theorems of moderate deviations, the asymptotic probabilities of small balls and so on. This work is motivated by researches on the record numbers.

### References

- [1] Chaumont, L. and Doney, R. A. (2010): Invariance principles for local times at the maximum of random walks and Lévy processes. *Ann. Probab.* 38, 1368-1389.
- [2] Tsirelson, Boris. (2013): From uniform renewal theorem to uniform large and moderate deviations for renewal- reward processes. *Electronic. Comm. Probab.* 18(52), 1-13.

- [3] Majumdar, S. N and Ziff, R. M. (2008): Universal record statistics of random walks and Lévy flights. *Phys.Rev. Lett.* 101: 050601.

## ERGODICITIES AND EXPONENTIAL ERGODICITIES OF BRANCHING PROCESSES WITH IMMIGRATION

Zenghu LI *Beijing Normal University, China*, E-mail: lizh@bnu.edu.cn

**Abstract:** Under natural assumptions, we prove the ergodicities and exponential ergodicities in Wasserstein and total variation distances of branching particle systems without or with immigration. The key of the approach is a set of estimates for the variations of the transition probabilities. The estimates in Wasserstein distance are derived from an upper bound of the kernels induced by the first moment of such a system. Those in total variation distance are proved by comparing the cumulant semigroup of the system with that of a continuous-state branching process. We also show a connection between the ergodicities of the associated immigration system and decomposable distributions.

## A MECHANICAL MODEL OF BROWNIAN MOTION INCLUDING LOW ENERGY LIGHT PARTICLES

Song LIANG *Waseda University, Japan*, E-mail: liang@waseda.jp

**Abstract:** Consider a system with one massive particle interacting with an ideal gas, composed of infinitely many light particles. As soon as the initial condition is given, the system evolves according to non-random Newton mechanical principles, via interaction potentials, with its Hamiltonian given by

$$\frac{1}{2}|V|^2 + \sum_{(x,v)} \frac{m}{2}|v|^2 + \sum_{(x,v)} U(x - X).$$

Here  $(X, V)$  is the (position, velocity) of the massive particle,  $m$  is the mass of a light particle,  $(x, v)$  is the (position, velocity) of each light particle, and  $U \in C_0^\infty$  is the potential function. We are interested in the limiting behavior of the massive particle when  $m$ , the mass of environmental light particles converges to 0, while the density and the velocities of them go to infinity in proper scale.

For the case where the initial energies of all light particles are high enough, it is proved that the (position, velocity)-process of the massive particle converges to a diffusion process. In this talk, we consider the case where there also exist light particles with their initial energies not high enough, and prove that under certain conditions, the same convergence holds.

## LONG-TIME BEHAVIORS OF MEAN-FIELD INTERACTING PARTICLE SYSTEMS RELATED TO MCKEAN-VLASOV EQUATION

Wei LIU *Wuhan University, China*, E-mail: wliu.math@whu.edu.cn

**Abstract:** In this talk, we will show the gradient estimate of the Poisson equation, the exponential convergence in the Wasserstein metric  $W_{1,d_{t_1}}$  and uniform in time propagation of chaos for the mean-field weakly interacting particle system related to McKean-Vlasov equation. By means of the known approximate componentwise reflection coupling and with the help of some new cost function, we obtain explicit estimates for those three problems, avoiding the technical conditions in the known results. Our results apply when the confinement potential  $V$  has many wells, the interaction potential  $W$  has bounded second mixed derivative  $\nabla_{xy}^2 W$  which should be not too big so that there is no phase transition. As application, we obtain the concentration inequality of the mean-field interacting particle system with explicit constant, uniform in time. Several examples are provided to illustrate these results.

## ON GEOMETRIC AND ALGEBRAIC TRANSIENCE FOR BLOCK-STRUCTURED MARKOV CHAINS

Yuanyuan LIU *Central South University, China*, E-mail: liuyy@csu.edu.cn

**Abstract:** Block-structured Markov chains model a large variety of queueing problems and have many important applications in various areas. Stability properties have been investigated well for these Markov chains. In this talk, we will present the transient properties for two specific types of block-structured Markov chains, including  $M/G/1$ -type Markov chains and  $GI/M/1$ -type Markov chains. The necessary and sufficient conditions in terms of system parameters are obtained for geometric transience and algebraic transience. Possible extensions of the results to continuous-time Markov chains are also considered.

This talk is based on the joint work with Li Wendi and Li Xiuqin.

## THE LIMIT DISTRIBUTION OF INHOMOGENEOUS MARKOV PROCESSES AND KOLOMOGOROV'S PROBLEM

Zhenxin LIU *Dalian University of Technology, China*, E-mail: zxliu@dlut.edu.cn

**Abstract:** In this talk, we will talk about the limit distribution of inhomogeneous Markov processes generated by SDEs. Meantime, we will also discuss the recent progress in Kolmogorov's problem on the limit behavior of stationary distributions of diffusion processes as the diffusion tends to zero.

## THE HITTING TIMES FOR MARKOV CHAINS

Yonghua MAO *Beijing Normal University, China*, E-mail: maoyh@bnu.edu.cn

**Abstract:** We will review the fundamental roles of the hitting times playing for Markov chains, including the well-known roles for recurrence and stationarity, the essential roles for quasi-stationarity and cutoffs, the criterion provided for the advantage of asymmetry.

## THE EVOLUTION TO EQUILIBRIUM OF SOLUTIONS TO NONLINEAR FOKKER-PLANCK EQUATIONS

Michael Röckner *Bielefeld University, DE*, E-mail: roeckner@math.uni-bielefeld.de

**Abstract:** The talk is about the so-called  $H$ -Theorem for a class of nonlinear Fokker–Planck equations which are of porous media type on the whole Euclidean space perturbed by a transport term. We first construct a solution in the sense of mild solutions on  $L^1$  through a nonlinear semigroup of contractions. Then we study the asymptotic behavior of the solutions when time tends to infinity. For a large class  $M$  of initial conditions we show their relative compactness with respect to local  $L^1$  convergence, while all limit points belong to  $L^1$ . Under an additional assumption we obtain that we in fact have convergence in  $L^1$ , if the initial condition is a probability density. The limit is then identified as the unique stationary solution in  $M$  to the nonlinear Fokker–Planck equation. This solution is thus an invariant measure of the solution to the corresponding distribution dependent SDE whose time marginals converge to it in  $L^1$ . It turns out that under our conditions the underlying nonlinear Kolmogorov operator is a (both in the second and first order part) nonlinear analog of the generator of a distorted Brownian motion. The solution of the above mentioned distribution dependent SDE can thus be interpreted as a “nonlinear distorted Brownian motion”. Our main technique for the proofs is to construct a suitable Lyapunov function acting nonlinearly on the path in  $L^1$ , which is given by the nonlinear contraction semigroup applied to the initial condition, and then adapt a classical technique of Pazy to our situation. This Lyapunov function is given by a generalized entropy function (which in the linear case specializes to the usual Boltzmann–Gibbs entropy) plus a mean energy part. Joint work with Viorel Barbu (Romanian Academy, Iasi).

## THE EXISTENCE OF OPTIMAL CONTROL FOR CONTINUOUS-TIME MARKOV DECISION PROCESSES IN RANDOM ENVIRONMENTS

Jinghai Shao *Tianjin University, China*, E-mail: shaojh@tju.edu.cn

**Abstract:** In this talk, we investigate the optimal control problem for continuous-time Markov decision processes with the random impact of the environment. We provide conditions to show the existence of optimal controls under finite-horizon criteria. Under appropriate conditions, the value function is continuous and satisfies the dynamic programming principle. These results are established by introducing some restriction on the regularity of the optimal controls and by developing a new compactification method for continuous-time Markov decision processes, which is originally used to solve the optimal control problem for jump-diffusion processes. This is a joint work with Kun Zhao.

## THE THREE-DIMENSIONAL GAUSSIAN PRODUCT INEQUALITY

Wei SUN *Concordia University, Canada*, E-mail: wei.sun@concordia.ca

**Abstract:** We prove the 3-dimensional Gaussian product inequality, i.e., for any real-valued centered Gaussian random vector  $(X, Y, Z)$  and  $m \in \mathbf{N}$ , it holds that  $\mathbf{E}[X^{2m}Y^{2m}Z^{2m}] \geq$



$\mathbf{E}[X^{2m}]\mathbf{E}[Y^{2m}]\mathbf{E}[Z^{2m}]$ . Our proof is based on some improved inequalities on multi-term products involving 2-dimensional Gaussian random vectors. The improved inequalities are derived using the Gaussian hypergeometric functions and have independent interest. As by-products, several new combinatorial identities and inequalities are obtained. This talk is based on a joint work with Ze-Chun Hu and Guolie Lan.

## QUEUEING NETWORKS WITH INTERCHANGABLE, COOPERATIVE SERVERS

**Chia-Li WANG** *National Dong Hwa University, Taiwan*, E-mail: cwang@gms.ndhu.edu.tw

**Abstract:** In open Kelly and Jackson networks, servers are assigned to individual stations, serving customers only where they are assigned. We investigate the performance of modified networks where servers cooperate. A server who would be idle at the assigned station will serve customers at another station, speeding up service there. We assume *interchangeable* servers; the service rate of a server at a station depends only on the station, not the server. This gives *work conservation*, which is used in various ways. We investigate three levels of server cooperation, from *full cooperation*, where all servers are busy when there is work to do, anywhere in the network, to *one-way cooperation*, where a server assigned to one station may assist a server at another, but not the converse.

We obtain the same stability conditions for each level and, in a series of examples, obtain substantial performance improvement with server cooperation, even when stations before modification are moderately loaded.

## HEAT KERNEL ESTIMATES FOR SYMMETRIC PURE JUMP DIRICHLET FORMS

**Jian WANG** *Fujian Normal University, China*, E-mail: jianwang@fjnu.edu.cn

**Abstract:** In this talk, we consider the following symmetric non-local Dirichlet forms of pure jump type on metric measure space  $(M, d, \mu)$ :

$$D(f, g) = \int_{M \times M} (f(x) - f(y))(g(x) - g(y)) J(dx, dy),$$

where  $J(dx, dy)$  is a symmetric Radon measure on  $M \times M \setminus \text{diag}$  that may have different scalings for small jumps and large jumps. Under general volume doubling condition on  $(M, d, \mu)$  and some mild quantitative assumptions that  $J(x, y)$  has a light jumping tails of polynomial decay. We establish stability results for upper bounds of heat kernel (resp. two-sided heat kernel estimates) in terms of the jumping kernels, the cut-off Sobolev inequalities, and the Faber-Krahn inequalities (resp. the Poincaré inequalities). As an application, we also obtain stable characterizations of the corresponding parabolic Harnack inequalities. This is based on a joint work with Zhen-Qing Chen and Takashi Kumagai.

## WEAK CONVERGENCE OF MARTINGALES AND ITS APPLICATION TO NONLINEAR COINTEGRATING MODEL

**Dongsheng WU** *University of Alabama in Huntsville, USA*, E-mail: dongsheng.wu@uah.edu

**Abstract:** In this talk, we provide a weak convergence result for a class of martingales. As an application, using the marked empirical processes, we develop a test of parametric specification in a nonlinear cointegrating regression model. This talk is based on a joint work with Qiyang Wang and Ke Zhu.

## TRANSPORTATION-INFORMATION INEQUALITY FOR DISTANCE TIME MARKOV CHAINS

**Liming WU** *Chinese Academy of Sciences, China*, E-mail: Li-Ming.Wu@math.univ-bpclermont.fr

**Abstract:** Using large deviation idea, we propose the transportation-information inequality for discrete time Markov chains, and prove it is equivalent to the concentration inequality in the reversible case, generalizing the previous work of Guillin-Leonard-Wu-Yao (PTRF08). The counterpart in the non-reversible case is also discussed. I will present several easy-to-check sufficient conditions for this new kind inequality and several examples.

## ON A LAW OF LARGE NUMBER FOR LAST PASSAGE PERCOLATION ON COMPLETE GRAPH

**Xianyuan WU** *Capital Normal University, China*, E-mail: wuxy@cnu.edu.cn

**Abstract:** Consider the Last Passage Percolation on complete graph with vertex set  $\{1, 2, 3, \dots, n\}$ . Suppose that the edge passage time  $X$  be light-tailed with tail probability  $H(x) = P(X > x) = \exp\{-\beta(x)\}$ . Let  $W_n$  be the maximum passage time between 1 and  $n$  through self-avoiding pathes. We report here that a new weak Law of Large Number for  $W_n$  is obtained as the following:

$$\frac{W_n}{n\beta^{-1}(\log n)} \rightarrow 1, \quad \text{in probability}$$

as  $n \rightarrow \infty$ . Where  $\beta^{-1}$  is the inverse of  $\beta$ .

## SCHAUDER'S ESTIMATE FOR NONLOCAL EQUATIONS WITH SINGULAR LÉVY MEASURES

**Mingyan WU** *Wuhan University, China*, E-mail: mywu@whu.edu.cn

**Abstract:** Basing on Littlewood-Paley's decomposition and heat kernel estimates of integral form, we establish the Schauder estimate for the following non-local equation in  $\mathbb{R}^d$  with Hölder coefficients:

$$\partial_t u = \mathcal{L}_{\kappa, \sigma}^{(\alpha)} u + b \cdot \nabla u + f, \quad u(0) = 0,$$

where  $\alpha \in (1/2, 2)$  and  $b : \mathbb{R}_+ \times \mathbb{R}^d \rightarrow \mathbb{R}$  is an unbounded local  $\beta$ -order Hölder function in  $x$ , and  $\mathcal{L}_{\kappa, \sigma}^{(\alpha)}$  is a non-local  $\alpha$ -stable-like operator with form:

$$\mathcal{L}_{\kappa, \sigma}^{(\alpha)} u(t, x) := \int_{\mathbb{R}^d} \left( u(t, x + \sigma(t, x)z) - u(t, x) - \sigma(t, x)z^{(\alpha)} \cdot \nabla u(t, x) \right) \kappa(t, x, z) \nu^{(\alpha)}(dz),$$

where  $\kappa : \mathbb{R}_+ \times \mathbb{R}^d \rightarrow \mathbb{R}$  is bounded from above and below,  $\sigma : \mathbb{R}_+ \times \mathbb{R}^d \rightarrow \mathbb{R}^d \otimes \mathbb{R}^d$  is a  $\gamma$ -order Hölder continuous function in  $x$  uniformly in  $t$ , and  $\nu^{(\alpha)}$  is a singular non-degenerate  $\alpha$ -stable Lévy measure. (This is a joint work with Zimo Hao and Guohuan Zhao.)

## REGIME-SWITCHING JUMP DIFFUSIONS WITH NON-LIPSCHITZ COEFFICIENTS AND COUNTABLY MANY SWITCHING STATES

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George Yin *Wayne State University, USA*

Chao Zhu *University of Wisconsin-Milwaukee, USA*

**KEY WORDS:** regime-switching jump diffusion, non-Lipschitz condition, Feller property, strong Feller property.

**MATHEMATICAL SUBJECT CLASSIFICATION:** 60J27, 60J60, 60J75, 60G51.

**Abstract:** This work focuses on a class of regime-switching jump diffusion processes, which is a two component Markov processes  $(X(t), \Lambda(t))$ , where  $\Lambda(t)$  is a component representing discrete events taking values in a countably infinite set. Considering the corresponding stochastic differential equations, our main focus is on treating those with non-Lipschitz coefficients. We first show that there exists a unique strong solution to the corresponding stochastic differential equation. Then Feller and strong Feller properties are investigated.

## ON MODERATE DEVIATIONS IN POISSON APPROXIMATION

**Aihua XIA** *the University of Melbourne, VIC 3010, Australia*, E-mail:

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**Abstract:** The tail behaviour of Poisson distribution is very different from that of normal distribution and the right tail probabilities of counts of rare events are generally better approximated by the moderate deviations in Poisson distribution. We demonstrate that the moderate deviations in Poisson approximation generally require an adjustment and, using Stein's method and coupling techniques in Markov processes, we establish error estimates of the moderate deviations in Poisson approximation. The paper complements the works of [Chen & Choi (1992), Barbour, Chen & Choi (1995), Chen, Fang & Shao (2013)]. The talk is based on a joint work with Qingwei Liu.

## FRACTAL DIMENSION RESULTS FOR LÉVY AND LÉVY-TYPE PROCESSES

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**Keywords:** LÉVY PROCESSES, LÉVY-TYPE PROCESSES, HAUSDORFF DIMENSION, PACKING DIMENSION, UNIFORM DIMENSION RESULTS.

**Abstract:** We present some recent results on fractal dimension properties of Lévy and Lévy-type processes, including those for operator semi-stable Lévy processes [Kern, Meerschaert and

Xiao (2018), Luks and Xiao (2018)], the uniform dimension results for the images [Sun, Xiao, Xu and Zhai (2017)], and the inverse images [Song, Xiao and Yang (2018), Park, Xiao and Yang (2019)]. These results provide solutions to several questions raised in Xiao (2004) for Lévy and Lévy-type processes.

**Acknowledgement:** The talk is based on joint works with several coauthors. This research is supported by grants from the National Science Foundation.

## References

- [1] P. Kern, M. M. Meerschaert and Y. Xiao (2018). Asymptotic behavior of semistable Lévy exponents and applications to fractal path properties. *J. Theoret. Probab.* **31**, 598–617.
- [2] T. Luks and Y. Xiao (2018). On multiple points of operator semi-stable Lévy processes. *J. Theoret. Probab.* <https://doi.org/10.1007/s10959-018-0859-4>.
- [3] H. Park, Y. Xiao, and X. Yang (2019) Uniform dimension results for the inverse images of symmetric Lévy processes. *Submitted*.
- [4] R. Song, Y. Xiao and X. Yang (2018). Inverse images of stable Lévy processes. *Electron. Comm. Probab.* **23**, no. 75, 10 pp.
- [5] X. Sun, Y. Xiao, L. Xu and J. Zhai (2017). Uniform dimension results for a family of Markov processes. *Bernoulli*, to appear.
- [6] Y. Xiao (2004). Random fractals and Markov processes. In: *Fractal Geometry and Applications: A Jubilee of Benoit Mandelbrot*, (Michel L. Lapidus and Machiel van Frankenhuysen, editors), pp. 261–338, American Mathematical Society.

## PRINCIPAL VALUES OF SOME INTEGRAL FUNCTIONALS OF FRACTIONAL BROWNIAN MOTION

Litan YAN *Donghua University, China*, E-mail: litanyan@dhu.edu.cn

**Abstract:** Let  $B^H$  be a fractional Brownian motion with Hurst index  $0 < H < 1$  and the weighted local time  $\mathcal{L}^H(x, t)$ . In this talk, we consider the existence of the limit

$$\kappa_t^{H,f}(a) := \lim_{\varepsilon \downarrow 0} \left( \int_0^t f(B_s^H - a) 1_{\{|B_s^H - a| > \varepsilon\}} ds^{2H} + \zeta_t^H(a, \varepsilon) \right), \quad a \in \mathbb{R}, t \geq 0$$

in  $L^2$  and almost surely, where  $f$  is not locally integrable and

$$\zeta_t^H(a, \varepsilon) := \mathcal{L}^H(a + \varepsilon, t)g(a + \varepsilon) - \mathcal{L}^H(a - \varepsilon, t)g(a - \varepsilon).$$

with  $g' = f$ . This limit (if it exists) can be called the principal value of the integral  $\int_0^t f(B_s^H - a) ds^{2H}$ . By using the obtained results we give the Itô and occupation type formulas including the principal value.

## ON THE EXTINCTION-EXTINGUISHING FOR A STOCHASTIC LOTKA-VOLTERRA TYPE POPULATION DYNAMICAL SYSTEM

Xu YANG *North Minzu University, Yinchuan, China*, Email: xuyang@mail.bnu.edu.cn.

**Abstract:** We study a two-dimensional process  $(X, Y)$  arising as nonnegative solution to stochastic differential equations both driven by independent Brownian motions and compensated spectrally positive Lévy random measures. Both processes  $X$  and  $Y$  can be identified as continuous-state nonlinear branching processes where the evolution of  $Y$  is negatively affected by  $X$ . Assuming that process  $X$  extinguishes, i.e. it converges to 0 but never dies out in finite time, and process  $Y$  converges to 0, we identify rather sharp conditions under which the process  $Y$  exhibits, respectively, the following behaviors: extinction with probability one, or extinguishing with probability one, or both extinction and extinguishing occurring with strictly positive probabilities. This is a joint work with Yanxia Ren, Jie Xiong and Xiaowen Zhou.

## STABILITY OF REGIME-SWITCHING PROCESSES UNDER PERTURBATION OF TRANSITION RATE MATRICES

**Chenggui YUAN** *Swansea University, UK*, E-mail: C.Yuan@Swansea.ac.uk

**Abstract:** This work is concerned with the stability of regime-switching processes under the perturbation of the transition rate matrices. Two kind of perturbations are studied: the size of the transition rate matrix is fixed, and only the values of entries are perturbed; the values of entries are perturbed, and the size of the transition matrix is changed. Moreover, the coefficients of the underlying systems could be regular or irregular. This is joint work with Jinghai Shao.

## WELL-POSEDNESS AND LARGE DEVIATIONS FOR 2-D STOCHASTIC NAVIER-STOKES EQUATIONS DRIVEN BY MULTIPLICATIVE LÉVY NOISE

**Jianliang ZHAI** *University of Science and Technology of China, China*, E-mail: Zhajl@ustc.edu.cn

**Abstract:** Under the classical local Lipschitz and one sided linear growth assumptions on the coefficients of the stochastic perturbations, we first obtain the existence and the uniqueness of a strong (in both the probabilistic and PDEs sense) solution to the 2-D Stochastic Navier-Stokes equations driven by multiplicative Lévy noise. Applying the weak convergence method for the case of the Poisson random measures, we establish a Freidlin-Wentzell type large deviation principle for the strong solution in PDE sense. This talk is based on joint work with Zdzisław Brzeźniak and Xuhui Peng.

## MODERATE DEVIATION PROBABILITIES FOR EMPIRICAL DISTRIBUTION OF THE BRANCHING RANDOM WALK

**Shuxiong ZHANG** *Beijing Normal University, China*, E-mail: shuxiong.zhang@qq.com

**Abstract:** Given a super critical branching random walk  $\{Z_n\}_{n \geq 0}$ , let  $Z_n(A)$  be the number of particles located in some interval  $A$ , at time  $n$ . In this work, we study the convergence rate of

$\mathbb{P}\left(\frac{Z_n(\sqrt{n}A)}{Z_n(\mathbb{R})} \geq \nu(A) + \Delta_n\right)$ , where  $\nu(\cdot)$  is a standard Gaussian measure,  $\Delta_n$  is a positive sequence tending to zero.

## QUADRATIC TRANSPORTATION COST INEQUALITIES UNDER UNIFORM DISTANCE FOR STOCHASTIC REACTION DIFFUSION EQUATIONS DRIVEN BY MULTIPLICATIVE SPACE-TIME WHITE NOISE

**Tusheng ZHANG** *University of Manchester and University of Science and Technology of China*, E-mail: tusheng.zhang@manchester.ac.uk

**Abstract:** In this talk, we will present a quadratic transportation cost inequality for solutions of stochastic reaction diffusion equations driven by multiplicative space-time white noise based on a new inequality we proved for the moments (under the uniform norm) of the stochastic convolution with respect to space-time white noise, which is of independent interest.

## ASYMPTOTIC BEHAVIOUR OF HEAVY-TAILED BRANCHING PROCESSES IN RANDOM ENVIRONMENTS

Wenming HONG *Beijing Normal University, China*, E-mail: wmhong@bnu.edu.cn  
**Xiaoyue ZHANG** *Beijing Normal University, China*, E-mail: zhangxiaoyue@mail.bnu.edu.cn

**Abstract:** Consider a heavy-tailed branching process (denoted by  $Z_n$ ) in random environments, under the condition which infers that  $\mathbb{E} \log m(\xi_0) = \infty$ . We show that (1) there exists no proper  $c_n$  such that  $\{Z_n/c_n\}$  has a proper, non-degenerate limit; (2) normalized by a sequence of functions, a proper limit can be obtained, i.e.,  $y_n(\bar{\xi}, Z_n(\bar{\xi}))$  converges almost surely to a random variable  $Y(\bar{\xi})$ , where  $Y \in (0, 1)$   $\eta$ -a.s.; (3) finally, we give necessary and sufficient conditions for the almost sure convergence of  $\left\{\frac{U(\bar{\xi}, Z_n(\bar{\xi}))}{c_n(\bar{\xi})}\right\}$ , where  $U(\bar{\xi})$  is a slowly varying function that may depend on  $\bar{\xi}$ .

## STOCHASTIC LAGRANGIAN PATH FOR LERAY SOLUTIONS OF 3-D NAVIER-STOKES EQUATIONS

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**Abstract:** In this paper we show the existence of stochastic Lagrangian particle trajectory for Leray's solution of 3D Navier-Stokes equations. More precisely, for any Leray's solution  $\mathbf{u}$  of 3D-NSE and each  $(s, x) \in \mathbb{R}_+ \times \mathbb{R}^3$ , we show the existence of weak solutions to the following SDE, which has a density  $\rho_{s,x}(t, y)$  belonging to  $\mathbb{H}_q^{1,p}$  provided  $p, q \in [1, 2)$  with  $\frac{3}{p} + \frac{2}{q} > 4$ :

$$dX_{s,t} = \mathbf{u}(s, X_{s,t})dt + \sqrt{2\nu}dW_t, \quad X_{s,s} = x, \quad t \geq s,$$

where  $W$  is a three dimensional standard Brownian motion,  $\nu > 0$  is the viscosity constant. Moreover, we also show that for Lebesgue almost all  $(s, x)$ , the solution  $X_{s,\cdot}^n(x)$  of the above

SDE associated with the mollifying velocity field  $\mathbf{u}_n$  weakly converges to  $X_{s,\cdot}(x)$  so that  $X$  is a Markov process in almost sure sense. (This is a joint work with Guohuan Zhao.)

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## EXISTENCE OF GEOMETRIC ERGODIC PERIODIC MEASURES OF STOCHASTIC DIFFERENTIAL EQUATIONS

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**Abstract:** Periodic measures are the time-periodic counterpart to invariant measures for dynamical systems and can be used to characterise the long-term periodic behaviour of stochastic systems. This paper gives sufficient conditions for the existence, uniqueness and geometric convergence of a periodic measure for time-periodic Markovian processes on a locally compact metric space in great generality. In particular, we apply these results in the context of time-periodic weakly dissipative stochastic differential equations, gradient stochastic differential equations as well as Langevin equations. We will establish the Fokker-Planck equation that the density of the periodic measure sufficiently and necessarily satisfies. Applications to physical problems shall be discussed with specific examples.

(This is a joint work with Chunrong Feng and Johnny Zhong).

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## $\varepsilon$ -NASH EQUILIBRIUM FOR JSQ MODELS

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**Abstract:** This talk is a continuation of my talk at the 13th Workshop on Markov Processes and Related Topics in Wuhan. We investigate an unobservable join-the-shortest-queue system with  $N$  nodes, in which each node has a dedicated arrival stream, and there is an additional extra arrival stream to balance the system load by routing customers to the shortest queue. A reward-cost structure is introduced to the system and our focus is to analyze customers' strategic behaviour. The equilibrium and social optimal strategies are derived for the unobservable mean field limit model. We show that the strategies obtained for the mean field model well approximate the finite  $N$  model.

This talk is based on joint work with Qihui BU, Qiwei LIU, and Jianshan TANG.

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