The 4th workshop on branching processes and related topics

ABSTRACTS

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Contents (List by the order of presentation)

ON POST-CRISIS INTEREST RATE MODELLING

Wolfgang RUNGGALDIER *University of Padova, Italy*, E-mail: runggal@math.unipd.it

KEY WORDS: Interest rates, multicurves, arbitrage-free dynamics

MATHEMATICAL SUBJECT CLASSIFICATION: 91G30

Abstract: The so-called financial crisis has deeply affected the traditional interest rate theory, in particular for what concerns the modelling of the dynamics of the Libor rate which is one of the main reference rates. After recalling some basic notions from interest rate theory, we discuss various possibilities of modelling, in an arbitrage-free way, the dynamics of the Libor rate in a post-crisis (multicurve) setting.

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A FORWARD INTEREST RATES MODEL BASED ON BRANCHING PROCESSES

Ying Jiao *Université Claude Bernard-Lyon 1, France*, Email: ying.jiao@univ-lyon1.fr
Chunhua Ma *Nankai University, China*, E-mail: mach@nankai.edu.cn
Simone Scotti *Université Paris-Diderot, France*, E-mail: scotti@math.univ-paris-diderot.fr
CARLO SGARRA *Politecnico di Milano, Italy*, E-mail: carlo.sgarra@polimi.it

Key words: Branching Processes, Forward Rates, Heath-Jarrow-Morton Model, Self-Exciting Structures, Random Fields.

Mathematical Subject Classification: 60J75, 60J80, 60J85, 62P05, 91G30.

Abstract: We propose and investigate a market model for forward interest rates prices, including most basic features exhibited by previous models and taking into account self-exciting properties. The model proposed extends Hawkes-type models by introducing a two-fold integral representation property. A Random Field approach was already exploited by Sornette and Santa Clara, who adopted a Brownian Sheet framework for describing the forward rates dynamics. The novelty contained in our approach consists in combining the basic features of both Branching Processes and Random Fields in order to get a realistic and parsimonious model setting. We discuss the no-arbitrage issue of the present modelling framework and its completeness. We outline a possible methodology for parameters estimation. We illustrate by graphical representation the main achievements of this approach.

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Local times for spectrally negative Lévy processes

Bo Lijun *Nankai University, China*
Xiaowen Zhou *Concordia University, Canada*, E-mail: xiaowen.zhou@concordia.ca

Key words: local time, spectrally negative Lévy process, scale function, Continuous-state nonlinear branching process

Mathematical Subject Classification: 60H10, 60J80

Abstract: Spectrally negative Lévy processes are Lévy processes with no positive jumps. We identify several joint Laplace transforms involving local times for spectrally negative Lévy processes. The Laplace transforms are expressed in terms of scale functions. We also point out an application of such results in the study of continuous-stat nonlinear branching processes. This talk is based on joint work in Li and Zhou (2017) and joint work in progress with Dawson, Foucart and Li.

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Local asymptotic properties for Cox-Ingersoll-Ross process with discrete observations

Mohamed BEN ALAYA *University of Rouen, Laboratory LMRS, CNRS, (UMR 6085), France*, E-mail: mohamed.ben-alaya@univ-rouen.fr
Ahmed Kebaier *University Paris 13, Laboratory LAGA, CNRS, (UMR 7539), France*
Ngoc Khue Tran *Pham Van Dong University, Department of Natural Science Education,Vietnam*

KEY WORDS: Cox-Ingersoll-Ross process; local asymptotic (mixed) normality property; local asymptotic quadraticity property; Malliavin calculus; parametric estimation; square root coefficient

MATHEMATICAL SUBJECT CLASSIFICATION: 60H07; 65C30; 62F12; 62M05

Abstract: We consider a Cox-Ingersoll-Ross process whose drift coefficient depends on unknown parameters. Considering the process discretely observed at high frequency, we prove the local asymptotic normality property in the subcritical case, the local asymptotic quadraticity in the critical case, and the local asymptotic mixed normality property in the supercritical case. To obtain these results, we use the Malliavin calculus techniques developed recently for CIR process by Alós et al. [1] and Altmayer et al. [2] together with the L^p-norm estimation for positive and negative moments of the CIR process obtained by Bossy et al. [5] and Ben Alaya et al. [3,4]. In this study, we require the same conditions of high frequency $\Delta_n \to 0$ and infinite horizon $n\Delta_n \to 0$ as in the case of ergodic diffusions with globally Lipschitz coefficients studied earlier by Gobet [6]. However, in the non-ergodic cases, additional assumptions on the decreasing rate of Δ_n are required due to the fact that the square root diffusion coefficient of the CIR process is not regular enough. Indeed, we assume $\frac{n\Delta_n^{\frac{3}{2}}}{\log(n\Delta_n)} \to 0$ for the critical case and $n\Delta_n^2 \to 0$ for the supercritical case.

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Asymptotic expansions in the central limit theorem for a branching Wiener process

Zhiqiang GAO *Beijing Normal University, China*, E-mail: gaozq@bnu.edu.cn
Quansheng LIU *Université de Bretagne-Sud, France*

Key words: Branching Wiener processes, asymptotic expansion, central limit theorem

Mathematical Subject Classification: 60J10, 60F05

Abstract: We consider a branching Wiener process in $\mathbb{R}^d$, in which particles reproduce as a super-critical Galton-Watson process and disperse according to a Wiener process. For $B \subset \mathbb{R}^d$, let $Z_n(B)$ be the number of particles of generation n located in B. Révész, Rosen and Shi (2005) obtained the finite order asymptotic expansion of the local limit theorem for the appropriately normalized counting measure $Z_n(\cdot)$ under the second moment condition on the offspring distribution. Here we show the finite order asymptotic expansion in the central limit theorem under a weaker moment condition of the form $\mathbb{E}X(\log X)^{1+\lambda} < \infty$.

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Total length of the genealogical tree for quadratic CB process

Hongwei BI *UIBE, China*, E-mail: bihw@uibe.edu.cn
Jean Francois Delmas *ENPC, France*

KEY WORDS: branching process, genealogical tree, lineage tree, time-reversal

MSC: 60J80, 92D25, 60G10, 60G55

Abstract: We prove the existence of the total length process for the genealogical tree of a population model with random size given by quadratic stationary continuous-state branching processes. For the one-dimensional marginal, the Laplace transform as well as the fluctuation of the corresponding convergence is also given. This result is to be compared with the one obtained by Pfaffelhuber and Wakolbinger for a constant size population associated to the Kingman coalescent. We also give a time reversal property of the number of ancestors process at all times, and a description of the so-called lineage tree in this model.

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MULTITYPE BRANCHING PROCESSES IN RANDOM ENVIRONMENT

Elena DYAKONOVA *Steklov Mathematical Institute and Novosibirsk State University, Russia*
Vladimir VATUTIN *Steklov Mathematical Institute and Novosibirsk State University, Russia*, E-mail: vatutin@mi.ras.ru
Vitali WACHTEL *Institut für Mathematik, Universität Augsburg, Germany*

KEY WORDS: multitype branching processes, random environment, survival probability, limit theorem

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80

Abstract: We consider a $p-$type Galton-Watson branching process $\mathbf{Z}(n) = (Z^{(1)}(n), ..., Z^{(p)}(n)),\ \ n = 0, 1, ...,$ in i.i.d. random environment. Let $M_n = (m_{ij}(n))_{i,j=1}^{p}$ be the mean matrix of the reproduction law for the particles of the n-th generation,

$$\mathbf{M}_{n,1} := \|M_n \cdots M_1\|$$

be the norm of products of matrices and let

$$\Lambda_{Lya} := \lim_{n\to\infty} \frac{1}{n}\mathbf{E}\left[\log \mathbf{M}_{n,1}\right]$$

be the upper Lyapunov exponent of the sequence $\{M_n, n = 0, 1, ...\}$ of i.i.d. random matrices. Suppose that there exists a number $b > 1$ such that

$$\frac{1}{b} \le \frac{\max_{i,j} m_{ij}(n)}{\min_{i,j} m_{kl(n)}} \le b$$

with probability 1. Let $\mathbf{e}_i$ be the $p-$dimensional vector whose i-th component is equal to 1 and the others are zeros, $\mathbf{0} = (0, ..., 0)$ be the $p-$dimensional vector all whose components are zeros.

We show that if $\Lambda_{Lya} = 0$ then, under some mild additional technical conditions, for any $i \in \{1, ..., p\}$ there exists a number $c_i \in (0, \infty)$ such that

$$\lim_{n\to\infty} \sqrt{n}\mathbf{P}\left(\mathbf{Z}(n) \ne \mathbf{0} | \mathbf{Z}(0) = \mathbf{e}_i\right) = c_i.$$

If $\Lambda_{Lya} < 0$ and

$$\lim_{n\to\infty} \frac{1}{n}\mathbf{E}\left[\mathbf{M}_{n,1} \log \mathbf{M}_{n,1}\right] < 0$$

then, under some additional conditions

(a) for any $i \in \{1, ..., p\}$ there exists a number $c_i \in (0, \infty)$ such that

$$\mathbf{P}\left(\mathbf{Z}(n) \ne \mathbf{0} \Big| \mathbf{Z}(0) = \mathbf{e}_i\right) \sim c_i (\mathbf{E}\mathbf{M}_{n,1})^n, \quad n \to \infty;$$

(b) for each $\mathbf{s} \in [0, 1]^p$, $\mathbf{s} \ne \mathbf{1}$

$$\lim_{n\to\infty} \mathbf{E}\left[\mathbf{s}^{\mathbf{Z}(n)} \Big| \mathbf{Z}(n) \ne \mathbf{0}; \mathbf{Z}(0) = \mathbf{e}_i\right] = \Phi_i(\mathbf{s}),$$

where $\Phi_i(\mathbf{s})$ is the probability generating function of a proper distribution on the set of p-dimensional vectors with nonnegative integer-valued components.

This work was supported by the Russian Science Foundation under the grant 17-11-01173 and was fulfilled in the Novosibirsk State University.

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BRANCHING PROCESSES IN SEMI-MARKOV RANDOM ENVIRONMENT

Nikolaos LIMNIOS *Université de Technologie de Compiègne, Sorbonne Universités, France*
E-mail: nlimnios@utc.fr
Elena Yarovaya *Lomonosov Moscow State University, Russia*

Key words: Branching process, semi-Markov process, diffusion approximation, near critical case

Mathematical Subject Classification: 60J89, 60K15, 60J60, 60K37

Abstract:

We consider near critical continuous-time Markov age-dependent branching processes in semi-Markov ergodic random environment in general state space case. The law of offspring and the law of lifetimes are functions of the random environment. For this kind of processes we obtain averaging and diffusion approximation results and discuss some particular cases. The averaging lies to a Markov branching standard processes. In contrast to the usual transform method, (i.e., generating function approach, Laplace transform, etc.) we present a new method to obtain diffusion approximations of such processes based on semi-Markov compensating operator convergence and semimartingale relative compactness. Moreover, we prove that the near critical condition is a necessary and sufficient condition for a diffusion approximation of a semi-Markov branching process to hold.

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Uniqueness problem for SPDEs from population models

Jie Xiong *Southern University of Science & Technology, Shenzhen, China.*, E-mail: xiongj@sustc.edu.cn
Key words: Stochastic partial differential equation, superprocess, Fleming-Viot process, distribution function, backward doubly stochastic differential equation, pathwise uniqueness.

Mathematical Subject Classification: Primary: 60H15, 60J68; Secondary: 60G57, 60H05

Abstract: This is a survey on the uniqueness of the solutions to stochastic partial differential equations (SPDEs) related to two measure-valued processes: superprocess and Fleming-Viot process which are given as rescaling limits of from population biology models. We summarize recent results for Konno-Shiga-Reimers' and Mytnik's SPDEs, and their related distribution-function-valued SPDEs. This talk is based on a paper jointly with Xu Yang.

EXISTENCE AND PATHWISE UNIQUENESS TO AN SPDE DRIVEN BY COLORED α-STABLE NOISES

Jie Xiong *Southern University of Science, China*
Xu YANG *North Minzu University, China*, E-mail: xuyang@mail.bnu.edu.cn

KEY WORDS: Stochastic partial differential equation, colored noise, stable, existence, pathwise uniqueness.

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60H15; secondary 60K37, 60F05.

Abstract: In this paper we establish the pathwise uniqueness to a stochastic partial differential equation with Hölder continuous coefficient driven by a colored α-stable noise. We also study a stochastic differential equation system driven by Poisson random measures.

A CBI process approach to Financial modelling

Ying Jiao *Universit Claude BernardLyon 1, France*, E-mail: ying.jiao@univ-lyon1.fr

Abstract: In this talk, we explain how to describe some recent phenomenons observed on financial markets by using the framework of continuous-state branching processes with immigration (CBI). We propose and investigate the so-called alpha-CIR model, which is a generalization of the well-known Cox-Ingersoll-Ross model with alpha-stable Lvy jumps. By adopting the CBI approach, we model the self-exciting property, or the clustering effect, in a realistic and parsimonious way and hence provide nice interpretations for some stylized facts on bond and energy markets. This talk is based on joint works with Chunhua Ma (Nankai University), Simone Scotti (Universit Paris Diderot - Paris 7) and Carlo Sgarra (Politecnico di Milano).

ALPHA-HESTON MODEL

Ying Jiao *Université Claude Bernard-Lyon 1, Institut de Science Financier et d'Assurances. France and Peking University, BICMR, Beijing, China.*
Chunhua Ma *Nankai University, School of Mathematical Sciences and LPMC, Tianjin, China.*
Simone SCOTTI *Université Paris Diderot, LPSM, France*, E-mail: scotti@lpsm.paris
Chao Zhou *National University of Singapore, Singapore*
...

KEY WORDS: alpha-stable branching processes, Heston model, implied volatility surface, moment explosion, Pareto law, stable-CIR, VIX

MATHEMATICAL SUBJECT CLASSIFICATION: 91G20, 91G80, 60G60

Abstract: We introduce a new stock or exchange rate model, called the α-Heston model, which is a natural extension of the standard Heston model (1993) by adding a jump part driven by a α-stable Lévy process in the variance process in the spirit of Dawson and Li (2006), Fu and Li (2010) and Li and Ma (2015). We deduce an explicit expression of the Laplace transform by using the fact that the model belongs to the class of affine processes. We determine explicitly the moment explosions of the stock S and the variance V processes. In particular, we show that $E[S_t^u] < \infty$ if and only if $u \in [0, 1]$ for all $t > 0$, that is the minimal set. Moment explosions are closely related to the shape of the implied volatility surface, see Lee (2004), showing that α-Heston model is the "smiled" allowed by theoretical constraints. We also characterize the distribution tails of variance process V showing that the right tails is controlled by the parameter α whereas the left one depends only on CIR usual parameters. We deduce the behavior of the implied volatility surface for option written on the variance and VIX process. We propose a new decomposition of cluster effect, in the same spirit of Li (2017), splitting on large jumps, we highlight some properties of this declustering procedure. We also discuss a VIX data analysis pointing out the self-exciting structure and the Pareto law of large increments.

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LIMITING DISTRIBUTION OF PARTICLES NEAR THE FRONTIER IN THE CATALYTIC BRANCHING BROWNIAN MOTION

Sergey Bocharov *Zhejiang University, China*, E-mail: bocharov@zju.edu.cn

Key words: branching Brownian motion, local times

Mathematical Subject Classification: 60J80

Abstract: We consider the model of binary branching Brownian motion with spatially-inhomogeneous branching rate $\beta\delta_0(\cdot)$, where $\delta_0(\cdot)$ is the Dirac delta measure and β is a positive constant.

It was previously shown in [1] and [2] that if R_t, $t \geq 0$ is the position of the rightmost particle then as $t \to \infty$ $\frac{R_t}{t} \to \frac{\beta}{2}$ almost surely and $P(R_t - \frac{\beta}{2}t \leq x) \to E\exp\{-M_\infty e^{-\beta x}\}$, where M_∞ is the almost sure limit of some martingale.

In this talk we show that the distribution of particles centred about $\frac{\beta}{2}t$ converges to a mixed Poisson point process according to the martingale limit M_∞.

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law of large numbers for supercritical superprocesses with non-local branching

Sandra Palau *Department of Mathematical Sciences, University of Bath, UK.*
Ting YANG *School of Mathematics and Statistics, Beijing Institute of Technology, China*, E-mail: yangt@bit.edu.cn

KEY WORDS: Superprocesses, non-local branching mechanisms, law of large numbers

MATHEMATICAL SUBJECT CLASSIFICATION: 60J68, 60J80

Abstract: In this talk, we consider the superprocesses which undertake both local and non-local branching. Under mild conditions, there exists a leading eigenvalue of the associated Schrödinger operator. The sign of this eigenvalue distinguishes between the cases where there is local extinction and exponential growth. When this leading eigenvalue is positive (supercritical), we establish the weak and strong law of large numbers for the superprocesses.

Coalescent results for diploid exchangeable population models

Matthias Birkner *Institut für Mathematik, Johannes-Gutenberg-Universität Mainz, Germany*
Huili Liu *Hebei Normal University, China*, E-mail: liuhuili1112@163.com
Anja Sturm *Institut für Mathematische Stochastik, Georg-August-Universität Göttingen, Germany*
Key words: Diploid population model, diploid ancestral process, coalescent with simultaneous multiple collisions

Mathematical Subject Classification: Primary: 60J17, 92D10; Secondary: 60J70, 92D25

Abstract: We consider diploid bi-parental analogues of Cannings models: in a population of fixed size N the next generation is composed of $V_{i,j}$ offspring from parents i and j, where $V = (V_{i,j})_{1\le i\neq j\le N}$ is a (jointly) exchangeable (symmetric) array. Every individual carries two chromosome copies, each of which is inherited from one of its parents. We obtain general conditions, formulated in terms of the vector of the total number of offspring to each individual, for the convergence of the properly scaled ancestral process for an n-sample of genes towards a (Ξ-)coalescent. This complements Möhle and Sagitov's (2001) result for the haploid case and sharpens the profile of Möhle and Sagitov's (2003) study of the diploid case, which focused on fixed couples, where each row of V has at most one non-zero entry.

We apply the convergence result to several examples, in particular to two diploid variations of Schweinsberg's (2003) model, leading to Beta-coalescents with two-fold and with four-fold mergers, respectively.

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Application of the coalescence problem to branching random walks

Jyy-I Hong *National Sun Yat-sen University, Taiwan*
JYY-I HONG *National Sun Yat-sen University, Taiwan*, E-mail: hongjyyi@gmail.com
Krishna B. Athreya *Iowa State University, U.S.A.*
...

Key words: branching random walks, branching processes, coalescence, supercritical, explosive

Mathematical Subject Classification: 60J80

Abstract:
We consider a rapidly-growing Galton-Watson branching process and pick two individuals in the current generation by simple random sampling without replacement and trace their lines of descent backward in time till they meet for the first time. We call the common ancestor of these chosen individuals at the coalescent time their most recent common ancestor. The coalescence problem is to investigate the limit behaviors of some characteristics of this most recent common ancestor such as its death time and its generation number. In this talk, we apply the results in the the coalescence problem to find the limit distribution of the positions of the particles in the branching random walks as well as some results in the discounted cases.

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Affine Processes for Multiple Curve Modeling

Christa Cuchiero *University of Vienna, Austria*
Claudio FONTANA *Paris Diderot University, France*, E-mail: fontana@math.univ-paris-diderot.fr
Alessandro Gnoatto *University of Verona, Italy*
Guillaume Szulda *Paris Diderot University, France*

Key words: Multiple yield curves, Libor rate, forward rate agreement, multiplicative spread, affine processes, branching processes.

Mathematical Subject Classification: 91B24, 91B70, 91G30, 60G60.

Abstract:

We propose a general and tractable framework under which all multiple yield curve modeling approaches (short rate, Libor market, or HJM) based on affine processes can be consolidated. We model a numéraire process and multiplicative spreads between Libor rates and simply compounded OIS rates as functions of an underlying affine process. Besides allowing for ordered spreads and an exact fit to the initially observed term structures, this general framework leads to tractable valuation formulas for caplets and swaptions and embeds all existing multi-curve affine models. Moreover, the proposed approach gives rise to new developments, such as a short rate type model driven by a Wishart process, for which we derive a closed-form pricing formula for caplets. We also construct and study a multi-curve model based on a branching process with tempered α-stable Lévy measure. The empirical performance of two simple specifications is illustrated by calibration to market data.

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Affine Volterra processes

Eduardo Abi Jaber *Université Paris-Dauphine/CEREMADE and AXA Investment Managers, Paris, France*
Martin Larsson *ETH Zurich, Switzerland*
Sergio PULIDO *ENSIIE/LaMME Paris–Evry, France*, E-mail: sergio.pulidonino@ensiie.fr

Key words: stochastic Volterra equations, Riccati–Volterra equations, affine processes, rough volatility.

Abstract: Motivated by recent advances in rough volatility modeling, we introduce affine Volterra processes, defined as solutions of certain stochastic convolution equations with affine coefficients. Classical affine diffusions constitute a special case, but affine Volterra processes are neither semimartingales, nor Markov processes in general. Nonetheless, their Fourier-Laplace functionals admit exponential-affine representations in terms of solutions of associated deterministic integral equations, extending the well-known Riccati equations for classical affine diffusions. Our findings generalize and clarify recent results in the literature on rough volatility.

SAMPLE PATHS OF CONTINUOUS-STATE BRANCHING PROCESSES WITH DEPENDENT IMMIGRATION

Zenghu LI *Beijing Normal University, China*, E-mail: lizh@bnu.edu.cn

KEY WORDS: continuous-state branching process, dependent immigration, stochastic integral equation, Poisson random measure, Yamada–Watanabe type condition

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 60H10, 60H20

Abstract: We are interested in a class of Markov processes, which we call *continuous-state branching processes with dependent immigration* (CBDI-processes). By dependent immigration we mean the immigration rate depends on the state of the population via some function. This kind of immigration was studied in Dawson and Li (2003) and Fu and Li (2004) for measure-valued diffusions and extended in Li (2011) to general branching and immigration mechanisms. In those references, the immigration models were constructed in terms of stochastic equations driven by Poisson point measures on some path spaces. The approach is essential since the uniqueness of the corresponding martingale problems are still unknown. In the references mentioned above, the immigration rate functions were assumed to be Lipschitz and the existence of some excursion laws of the corresponding measure-valued branching processes without immigration was required.

The main result we present is a construction of the CBDI-process using a stochastic equation of the type of Li (2011), but with non-Lipschitz immigration rate functions. The point of the work is it gives a direct construction of the sample path of the CBDI-process with general branching and immigration mechanisms from those of the corresponding CB-process without immigration. By choosing the ingredients suitably, we can get either a new CB-process with different branching mechanism or a branching model with competition. A special case of the construction was given in the recent work of Li and Zhang (2018+) by tightness and weak convergence arguments, which require the existence of the second moment and the excursion law for the corresponding CB-process. We replace the second moment assumption by the first moment one and remove the assumption on the existence of the excursion law. Our approach gives a strong convergence of the iteration sequence of the solution. In the stochastic equation considered here, the continuous immigration is represented by a Poisson point measure based on a Kuznetsov measure of the CB-process, which is slightly different from the equation in Li and Zhang (2018+).

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Asymptotic Behaviour of Extinction Probability of Interacting Branching Collision Processes

Li Junping *Central South University, China*
Key words: Markov branching processes; Collision branching processes; Interaction

Mathematical Subject Classification: 60J27 60J35

Abstract: We consider the uniqueness and extinction properties of the Interacting Branching Collision Process (IBCP), which consists of two strongly interacting components: an ordinary Markov branching process (MBP) and a collision branching process (CBP). We establish that there is a unique IBCP, and derive necessary and sufficient conditions for it to be non-explosive that are easily to be checked. Explicit expressions are obtained for the extinction probabilities for both regular and irregular cases. The associated expected hitting times are also considered. Examples are provided to illustrate our results.

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A time-change for a stationary branching process

Romain Abraham *MAPMO, Université d'Orléans, France*
Jean-François Delmas *CERMICS, Université Paris-Est, France*
Hui HE *Beijing Normal University, China*, E-mail: hehui@bnu.edu.cn

Abstract: We consider a stationary branching process obtained by a subcritical continuous state branching process with immigration. We shall show that the reduced tree of the process, after a time-change, is a continuous time Galton-Watson tree with immigration. As a by-product, we get that in the stable setting the sizes of all families at any time induce a Poisson-Kingman partition, which forms a Poisson-Dirichlet distribution in the quadratic case.

Prolific Skeleton decompositions and a Ray-Knight type theorem for supercritical Lvy trees

Gernimo, Uribe Bravo *Universidad Nacional Automoma de Mexico, Mexico* E-mail: geronimo@matem.unam.mx

Abstract:

We first examine a chronological model on trees which generalizes the splitting trees introduced by Lambert in 2010. These trees are intimately linked to Lvy processes and can be either compact or locally compact and are formalized as Totally Ordered and Measured (TOM) trees. The locally compact TOM trees will be decomposed in terms of their prolific skeleton (consisting of its infinite lines of descent). When applied to splitting trees, this implies the construction of the supercritical ones (which are locally compact) in terms of the subcritical ones (which are compact) grafted onto a Yule tree (which corresponds to the prolific skeleton). As a second (related) direction, we study the genealogical tree associated to our chronological construction. This is done through the technology of the height process introduced by [DLG02]. In particular we prove a Ray-Knight type theorem which extends the one for (sub)critical Lvy trees to the supercritical case.

The one-dimensional KPP-equation driven by space-time white noise

Sandra KLIEM *Goethe University Frankfurt, Germany*, E-mail: kliem@math.uni-frankfurt.de

KEY WORDS: stochastic PDE; KPP equation; initial conditions with compact support; travelling wave; complete convergence

MATHEMATICAL SUBJECT CLASSIFICATION: 60H15, 35R60

Abstract: The one-dimensional KPP-equation driven by space-time white noise,

$$\partial_t u = \partial_{xx} u + \theta u - u^2 + u^{\frac{1}{2}} dW, \qquad t > 0, x \in \mathbb{R}, \theta > 0, \qquad\qquad u(0,x) = u_0(x) \geq 0$$

is a stochastic partial differential equation (SPDE) that exhibits a phase transition for initial non-negative finite-mass conditions. Solutions to this SPDE can be seen as the high density limit of particle systems which undergo branching random walks with an extra death-term due to competition or overcrowding. They arise for instance as (weak) limits of approximate densities of occupied sites in rescaled one-dimensional long range contact processes.

If θ is below a critical value θ_c, solutions die out to 0 in finite time, almost surely. Above this critical value, the probability of (global) survival is strictly positive. Let $\theta > \theta_c$, then there exist stochastic wavelike solutions which travel with non-negative linear speed. For initial conditions that are "uniformly distributed in space", the corresponding solutions are all in the domain of attraction of a unique non-zero stationary distribution.

In my talk, I will introduce the model in question and give an overview of existing results, open questions and techniques involved in its analysis.

Generators of measure-valued jump-diffusions

Martin LARSSON *ETH Zurich, Switzerland*, E-mail: martin.larsson@math.ethz.ch
Sara Svaluto-Ferro *University of Vienna, Austria*

KEY WORDS: Measure-valued jump-diffusions, mean-field interaction, branching processes

MATHEMATICAL SUBJECT CLASSIFICATION: 60J68, 60J80

Abstract: Measure-valued jump-diffusions are useful as tractable approximations of large but finite stochastic systems, for instance large sets of equity returns. As in the finite-dimensional case, one can define an associated operator that reflects the dynamics of the system. We show that, in full analogy with the finite-dimensional case, such operators are of Lévy type expressed using a notion of derivative that is well-known from the superprocess literature. Examples falling into this framework include large population limits of particle systems with mean-field interaction, as well as measure-valued polynomial diffusions such as Fleming-Viot processes. The latter class encompasses a broad range of specifications that retain computational tractability. We also discuss further applications such as optimal control of measure-valued jump-diffusions.

Aldous move on cladograms in the diffusion limit

Wolfgang Löhr *TU Chemnitz & Universität Duisburg-Essen, Germany*
Leonid Mytnik *Technion Haifa, Israel*
Anita Winter *Universität Duisburg-Essen, Germany*, E-mail: anita.winter@uni-due.de

Key words: algebraic trees, metric trees, tree-valued Markov chains

Mathematical Subject Classification: 60G09, 60B05, 60J80

Abstract: In this talk we are interested in limit objects of graph-theoretic trees as the number of vertices goes to infinity. Depending on which notion of convergence we choose different objects are obtained.

One notion of convergence with several applications in different areas is based on encoding trees as metric measure spaces and then using the Gromov-weak topology. Apparently this notion is problematic in the construction of scaling limits of tree-valued Markov chains whenever the metric and the measure have a different scaling regime. We therefore introduce the notion of algebraic measure trees which capture only the tree structure but not the metric distances. Convergence of algebraic measure trees will then rely on weak convergence of the random shape of a subtree spanned a sample of finite size.

We will be particularly interested in binary algebraic measure trees which can be encoded by triangulations of the circle. We will show that in the subspace of binary algebraic measure trees sample shape convergence is equivalent to Gromov-weak convergence when we equip the algebraic measure tree with an intrinsic metric coming from the branch point distribution.

The main motivation for introducing algebraic measure trees is the study of a Markov chain arising in phylogeny whose mixing behavior was studied in detail by Aldous (2000) and Schweinsberg (2001). We give a rigorous construction of the diffusion limit as a solution of a martingale problem and show weak of the Markov chain to this diffusion as the number of leaves goes to infinity.

Near critical percolation of mean-field graphs

Olivier Hénard *LMO, Université Paris-Sud*

Key words: Percolation, Graphs, Branching Processes.

Mathematical Subject Classification: 60K35 60C05 05C80

Abstract: We discuss the subcritical percolation of (certain) large vertex-transitive graphs. Under a natural assumption, reminiscent of the one of Nachmias, we prove a sharp result on the size of the largest components. We also discuss a related result for the diameter (defined as the maximal diameter among the connected components), and point out that, in our regime, the component with the largest number of vertices is *not* the one with the largest diameter. Our results have certain implications on the weak supercritical regime. Graphs satisfying our assumption are, apart from the complete graph (the percolation of which is the Erdös–Rényi random graph), the cartesian product of 2 or 3 complete graphs and expanders graphs. A common feature of these graphs is that, if d stands for the common degree of the vertices, the percolation probability $1/(d-1)$ lies in the critical window: this allows the branching approximation process to be relevant.

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Branching processes in finance: LIBOR models with positive rates and spreads

Antonis Papapantoleon *National Technical University of Athens, Greece*, E-mail: papapan@math.ntua.gr

Key words: Branching processes, affine processes, LIBOR, positive rates, positive spreads, analytically tractable models.

Mathematical Subject Classification: 91G30

Abstract: The London Interbank Offered Rate (LIBOR) is a very important quantity in modern financial markets, as numerous financial products and transactions are associated to the daily fixing of this rate. Therefore, the modeling of the LIBOR has attracted significant attention from practitioners and academics interested in mathematical finance. The standard models for the LIBOR suffer from the well-known problem, that either the model is analytically tractable but the rates can be negative, or that the rate is positive but the model is not tractable. The first part of this talk will show how branching processes can be utilized to create a LIBOR model that is both analytically tractable and produces positive rates. The second part will discuss extensions of this model to the multiple curve and negative rates framework, as well as the computation of value adjustments in this class of LIBOR models.

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Title：TBD

Martino, Grasselli, Università degli Studi di Padova, Italy,

Email：grassell@math.unipd.it

Asymptotic behaviors of record numbers in simple random walks

Qiang YAO *School of Statistics, East China Normal University*, E-mail: qyao@sfs.ecnu.edu.cn

Key words: Random walk; laws of large numbers; Green's function; large deviations

Mathematical Subject Classification: 60F17; 60J80

Abstract: The study of record numbers in Markov Chains has been an important topic in the literature of probability, statistics, and physics. Motivated by the asymptotic behaviors of records in branching random walks, we investigate several important asymptotic behaviors of record numbers in one dimensional simple random walks, including laws of large numbers, large deviations, etc. Then we connect our results to the case of branching random walks. Joint work in progress with Yuqiang LI.

Some properties of a max-type recursive model

Xinxing Chen, *Shanghai Jiaotong University, China,* chenxinx@sjtu.edu.cn

Abstract: We consider a simple max-type recursive model which was introduced in the study of depinning transition in presence of strong disorder by Derrida and Retaux. Our interest is focused on the critical regime, for which we study the extinction probability and the moment generating function. This talk is based on a joint work with Bernard Derrida, Yueyun Hu, Mikhail Lifshits and Zhan Shi.

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Spine Decompositions and Limit Theorems for Critical Superprocesses

Yan-Xia Ren *Peking University, China*, E-mail: yxren@math.pku.edu.cn
Renming Song *University of Illinois at Urbana-Champaign, USA*
Zhenyao Sun *Peking University, China*

Key words: Critical superprocess, size-biased Poisson random measure, spine decomposition, 2-spine decomposition, asymptotic behavior of the survival probability, Yaglom's exponential limit law, martingale change of measure

Mathematical Subject Classification: 60J80, 60F05

Abstract: We first establish a spine decomposition theorem and a 2-spine decomposition theorem for some critical superprocesses. These two kinds of decompositions are unified as a decomposition theorem for size-biased Poisson random measures. Then we use these decompositions to give probabilistic proofs of the asymptotic behavior of the survival probability and Yaglom's exponential limit law for some critical superprocesses.

Low-dimensional lonely branching random walks die out

Rongfeng Sun *National University of Singapore, Singapore*

Abstract: The lonely branching random walks on Z^d is an interacting particle system where each particle moves as an independent random walk and undergoes critical binary branching when it is alone. We show that if the symmetrized walk is recurrent, lonely branching random walks die out locally. Furthermore, the same result holds if additional branching is allowed when the walk is not alone. Joint work with Matthias Birkner (U. Mainz).

Kingman Coalescent and Bayesian Analysis

SHUI FENG *McMaster University, Canada*, E-mail: shuifeng@mcmaster.ca

KEY WORDS: Bayesian nonparametrics, Dirichelt process, Fleming-Viot process, Kingman coalescent...

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 62C10; secondary 62M05

Abstract: Consider a population of individuals of various types. The type frequencies evolve according the Fleming-Viot process with parent independent mutation ([1]). Ancestral inference can be done through Kingman coalescent based on random samples taking at each fixed time. The focus of this talk is on making ancestral inferences for unobserved samples based on ancestral inference of the observed samples. This leads to the interplay between the Bayesian nonparametric analysis and Kingman coalescent. The talk is based on a joint work with Stefano Favaro and Paul Jenkins ([2]).

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Title：TBD

Chunhua Ma, Nankai University, China *Email：mach@nankai.edu.cn*

ALMOST SURE AND L_1-GROWTH BEHAVIOR OF SUPERCRITICAL MULTI-TYPE CBI PROCESSES

Mátyás BARCZY *MTA-SZTE Analysis and Stochastics Research Group, University of Szeged, Hungary*
E-mail: barczy@math.u-szeged.hu
Sandra Palau *University of Bath, United Kingdom*
Gyula Pap *University of Szeged, Hungary*

Key words: multi-type CBI process, almost sure and L_1-growth behaviour

Mathematical Subject Classification: 60J80, 60F15

Abstract: Under a first order moment condition on the immigration mechanism, we show that an appropriately scaled supercritical and irreducible multi-type continuous state and continuous time branching process with immigration (CBI process) converges almost surely. If an $x\log(x)$ moment condition on the branching mechanism does not hold, then the limit is zero. If this $x\log(x)$ moment condition holds, then we prove L_1 convergence as well. The projection of the limit on any left non-Perron eigenvector of the branching mean matrix is vanishing. If, in addition, a suitable extra power moment condition on the branching mechanism holds, then we provide the correct scaling for the projection of a CBI process on certain left non-Perron eigenvectors of the branching mean matrix in order to have almost sure and L_1 limit. A representation of the limits is also provided under the same moment conditions. Our results extend some results of Kyprianou, Palau and Ren [2] to certain left non-Perron eigenvectors of the branching mean matrix of the multi-type CBI process in question. The full presentation of our results can be found in [1].

In his talk Gyula Pap will prove asymptotic mixed normality of an appropriately scaled projection of a supercritical and irreducible multi-type CBI process on certain left non-Perron eigenvectors of the branching mean matrix under a second order moment condition on the branching and immigration mechanisms.

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ASYMPTOTIC BEHAVIOR OF PROJECTIONS OF SUPERCRITICAL MULTI-TYPE CBI PROCESSES

Gyula PAP *University of Szeged, Hungary* E-mail: papgy@math.u-szeged.hu
Mátyás Barczy *MTA-SZTE Analysis and Stochastics Research Group, University of Szeged, Hungary*
Sandra Palau *University of Bath, United Kingdom*

Key words: multi-type CBI process, mixed normal distributions

Mathematical Subject Classification: 60J80, 60F15

Abstract:

This talk is a continuation of the talk of Mátyás Barczy, who provides the correct scaling for the projection of a supercritical and irreducible multi-type continuous state and continuous time branching process with immigration (CBI process) on certain left non-Perron eigenvectors of the branching mean matrix in order to have almost sure and L_1 limit.

Under a second order moment condition on the branching and immigration mechanisms, we show that an appropriately scaled projection of a supercritical and irreducible multi-type CBI process on the other left non-Perron eigenvectors of the branching mean matrix is asymptotically mixed normal. In case of a non-vanishing immigration, with an appropriate random scaling, we prove asymptotic normality as well. Note that these results can be considered as counterparts of Theorems 3 and 4 and Corollaries 1 and 2 in Section 8 in Chapter V in Athreya and Ney [1] (which are derived for multidimensional discrete state continuous time Markov branching processes without immigration). In case of a non-vanishing immigration, we prove the almost sure convergence of the relative frequencies of distinct types of individuals as well.

References

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