

随机过程暑期学校（2017年7月2日-7月9日）

演讲人

陈木法 北京师范大学
陈昕昕 里昂一大（University Lyon 1）
洪文明 北京师范大学
胡跃云 巴黎十三大（Université Paris XIII）
李增沪 北京师范大学
刘全升 南布列塔尼大学（Université de Bretagne-Sud）
马春华 南开大学
马宇韬 北京师范大学
毛永华 北京师范大学
邵井海 北京师范大学
熊捷 澳门大学
王凤雨 北京师范大学
王颖喆 北京师范大学
张梅 北京师范大学
张余辉 北京师范大学
周晓文 康考迪亚大学（Concordia University）

地点：上午后主楼 1223 报告厅，下午后主楼 1129 报告厅

7月2日

8: 30--9: 30 陈木法
9: 50—11: 30 马春华（2 课时）
2: 30—5: 15 陈昕昕（3 课时）

7月3日

8: 30—11: 15 马春华（3 课时）
2: 30—5: 15 陈昕昕（3 课时）

7月4日

8: 30—10: 10 胡跃云（2 课时）
10: 20—12: 00 李增沪
2: 30—4: 10 熊捷（2 课时）
4: 20—6: 00 邵井海

7月5日

8: 30—10: 10 熊捷（2 课时）
10: 20—12: 00 毛永华
2: 30—4: 10 胡跃云（2 课时）
4: 20—6: 00 王颖喆

7月6日

8: 30—10: 10 熊捷 (2 课时)
10: 20—12: 00 马宇韬
2: 30—4: 10 胡跃云 (2 课时)

7 月 7 日

8: 30—10: 10 刘全升 (2 课时)
10: 20—12: 00 王凤雨
2: 30—4: 10 周晓文 (2 课时)
4: 20 - 6: 00 张梅

7 月 8 日

8: 30—10: 10 刘全升 (2 课时)
10: 20—12: 00 洪文明
2: 30—4: 10 周晓文 (2 课时)

7 月 9 日

8: 30—10: 10 周晓文 (2 课时)
10: 20—12: 00 张余辉
2: 30—4: 10 刘全升 (2 课时)

摘要

Branching processes and the associated martingales

Xinxin Chen (陈昕昕)

1. GW processes and Kesten-Stigum theorem
2. Branching random walk and Biggin's theorem
3. Multitype branching processes and Biggins-Kyprianou theorem

随机游动中的分枝机构及其应用

洪文明

通过分析随机游动的轨道, 得到其中的分枝过程; 在紧邻的情形, 这一结果可以追溯到 Harris (1952); 2. 介绍 Kesten, H., Kozlov, M. V., Spitzer, F. (1975, *Compositio Math.*) 将这一结果应用于随机环境中的随机游动 (RWRE) 的研究, 得到深刻的极限定理 stable law; 3. 介绍我们最近的工作, 将这些结果拓展到了有界跳幅的情形。

Randomly biased walks on trees

Yueyun Hu (胡跃云)

Random walk in random environment (RWRE), or more generally the subject of random media, has been a subject of much interest and efforts in probability over the last three decades. We discuss here a specific model of RWRE introduced by Lyons and Pemantle (1992), called randomly biased walks on trees, which is a model of reversible random walks in an infinite dimensional space. The two characteristics (reversibility and tree structure) make the model much richer than a simple extension of a one-dimensional RWRE. This model is naturally related to the studies of two other important models: the so-called trap model (see e.g. Ben Arous and Cerny (2007)) and the branching random walks. The link with trap models plays a crucial role in various interesting properties such as scaling limits and aging phenomenon, see e.g. Ben Arous and Hammond (2012). The branching random walks, whose studies were initialized by Hammersley, Kingman and Biggins (1974--1976), have received many recent developments, see e.g. Shi (2015) Saint-Flour's course for general references.

The main objective of this mini-course is the exploration of the link between a class of randomly biased walks on trees and some tree-indexed processes (branching random walks, Mandelbrot's cascades, ...). The outline is as follows:

1. Lyons and Pemantle's criterion on the recurrence/transience.
2. The sub-diffusive regime and Mandelbrot's cascades.
3. The slow regime and branching random walks in the boundary case.
4. Maximum of local times and a multi-type branching process.

Continuous state branching processes and related topics

李增沪, 马春华

经典的分枝过程主要包括离散状态 Galton-Watson 分枝过程 (GW 过程)、连续状态分枝过程 (CB 过程) 以及相应的移民过程。首先我们介绍 GW 过程并简要回顾 GW 过程一些基本的结果: 如灭绝概率, Yaglom 平稳分布, Q-过程等。在此基础上我们转向 CB 过程。

CB 过程可看作 GW 过程的重整化极限 (改变时间尺度并令粒子质量以适当方式趋于 0)。这里我们重点介绍 CB 过程的两种构造: 生成元法, 随机微分方程法。并通过简述 (或计算) CB 过程的一些基本性质来进一步体会上述两种构造方法。

参考文献:

Li, Z.H. (2012) : Continuous-state branching processes. Arxiv 1202.3223.

Title: limit theorems for branching processes in random environments

Quansheng Liu (刘全生)

I will present recent results about the asymptotic properties of a single type branching process (Z_n) in a random environment. In particular, I will present large deviation principles and expansions on $\log Z_n$, a Berry-Esseen bound in a central limit theorem, asymptotic properties of the harmonic moments of Z_n and of its normalized limit W , as well as those of the distribution $P(Z_n=k)$.

DISTANCES BETWEEN RANDOM ORTHOGONAL MATRICES AND INDEPENDENT NORMALS

马宇韬

Let Γ_n be an $n \times n$ Haar-invariant orthogonal matrix. Let Z_n be the $p \times q$ upper-left submatrix of Γ_n , where $p = pn$ and $q = qn$ are two positive integers. Let G_n be a $p \times q$ matrix whose pq entries are independent standard normals. In this paper we consider the distance between $\sqrt{n}Z_n$ and G_n in terms of the total variation distance, the Kullback-Leibler distance, the Hellinger distance and the Euclidean distance. We prove that each of the first three distances goes to zero as long as pq/n goes to zero, and not so if (p, q) sits on the curve $pq = \sigma n$, where σ is a constant. However, it is different for the Euclidean distance, which goes to zero provided pq^2/n goes to zero, and not so if (p, q) sits on the curve $pq^2 = \sigma n$. A previous work by Jiang [17] shows that the total variation distance goes to zero if both p/n and q/n go to zero, and it is not true provided $p = c n^{1/2}$ and $q = d n^{1/2}$ with c and d being constants. One of the above results confirms a conjecture that the total variation distance goes to zero as long as $pq/n \rightarrow 0$ and the distance does not go to zero if $pq = \sigma n$ for some constant σ .

马氏链的拟平稳性

毛永华

(1) 马氏链的拟平稳性的历史与发展, 包括 Yaglom 极限, 有限马氏链, 可数状态马氏链; 分支过程和生灭过程; (2) 三个基本问题和研究现状, 包括存在性, 吸引域和收敛速度, Lyapunov 条件判定, 在其他领域的应用; (3) (我们的) 进展, 包括爆炸的生灭过程和扩散过程; 线性生灭过程的吸引域。

题目：随机环境下扩散过程的遍历性、稳定性

邵井海

介绍用跳过程刻画的随机环境下，如何判断扩散过程的遍历性和稳定性。从过程的存在唯一性出发，介绍常返、非常返、指数遍历、强遍历等的判别准则。并且，用几何布朗运动，捕食模型等简单的例子说明此种随机环境下扩散过程的特点。

题目：变测度耦合方法与应用

王凤雨

为建立热半群的无穷维 Harnack 不等式，引入马氏过程的变测度耦合方法，并将该方法应用于建立随机(偏)微分方程的 Bismut 导数公式、Driver 分部积分公式，进而获得热核估计、传输不等式、强 Feller 性以及不变概率测度的刻画等。以简单的随机微分方程为例，介绍如何针对不同的问题，构造相应的变测度耦加以研究。

模拟退火算法与非齐次马氏过程

王颖喆

介绍关于非齐次马氏链的性质以及在随机智能计算中的应用，其中包括非齐次马氏链和随机智能算法（模拟退火）的各种收敛性的判定。

Backward stochastic differential equations and stochastic partial differential equations

Jie Xiong (熊捷)

We will first introduce a motivating example from mathematical finance where the backward stochastic differential equation (BSDE) arises naturally. We will then introduce the basic results for BSDEs including its relationship with a nonlinear partial differential equation (PDE). We will also introduce the basic results of stochastic PDE and its relationship with a class of BSDE.

题目：带移民的分枝过程

张梅

首先我们介绍带移民分枝过程的构造，然后介绍带移民分枝过程的有关极限性质，文献的一些研究进展以及我们的工作。

题目：从单生到单死

张余辉

单生过程的流出边界至多一个极点，所以成为诸多经典问题的显式判别准则可以期望的最大一类过程。事实也是如此，单生过程的研究成果丰富且相对完整，在这个报告里，我们介绍这些成果，同时介绍其 Poisson 方程解的显式表示以及由此得到的各种问题的统一处理方法。

分枝过程是单死过程的特殊一类，其理论丰富且应用广泛，但对一般的单死过程，我们所知有限。这里我们“对偶”地介绍单死过程的一些研究进展，包括 Siegmund 对偶，零流入和平均击中时等。

CMJ branching processes coded by spectrally positive Levy processes

Xiaowen Zhou (周晓文)

A homogeneous Crump–Mode–Jagers (CMJ) branching process is a general branching process with constant branching rate and with general life time distribution. It is known that a binary homogeneous CMJ process is associated to a local time of spectrally positive Levy process with sample paths of bounded variation; see Lambert (2010). In the lectures, we first go over basic facts of spectrally one-sided Levy processes such as fluctuation identities, Wiener-Hopf factorization, scale functions and solution to the exit problems. We then present the CMJ branching process coded by a spectrally positive Levy excursion. Using the fluctuation results for Levy processes we can carry out some computations on the binary homogeneous CMJ branching process.