

**The 7th Workshop on  
MARKOV PROCESSES AND RELATED TOPICS**

July 19 - 23, 2010

**No.6 Lecture room on 3th floor, Jingshi Building (京师大厦)  
Beijing Normal University**

**Organizers:** Mu-Fa Chen, Zeng-Hu Li, Feng-Yu Wang

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	July 19	July 20	July 21	July 22	July 23
Chairman	Mu-Fa Chen	Fu-Zhou Gong	Shui Feng	Jie Xiong	Dayue Chen
Speaker	Opening 8:30–8:40	Shuenn-Jyi Sheu 8:30–9:00	Anyue Chen 8:30–9:00	Leonid Mytnik 8:30–9:00	Zengjing Chen 8:30–9:00
	M. Fukushima 8:40–9:30	Feng-Yu Wang 9:00–9:30	Jiashan Tang 9:00–9:30	Quansheng Liu 9:00–9:30	Dong Han 9:00–9:30
	Tea Break				
	Zhen-Qing Chen 10:00–10:30	Renming Song 10:00–10:30	Xia Chen 10:00–10:30	Hao Wang 10:00–10:30	Zongxia Liang 10:00–10:30
	Panki Kim 10:30–11:00	Christian Leonard 10:30–11:00	Yutao Ma 10:30–11:00	Xiaowen Zhou 10:30–11:00	Fubao Xi 10:30–11:00
	Jinghai Shao 11:00–11:30	Xu Zhang 11:00–11:20	Liang-Hui Xia 11:00–11:20	Hui He 11:00–11:20	Liqun Niu 11:00–11:20
Lunch					
Chairman	Feng-Yu Wang	Shizan Fang		Zengjing Chen	Zeng-Hu Li
Speaker	Chii-Ruey Hwang 14:30–15:00	Tusheng Zhang 14:30–15:00		Dayue Chen 14:30–15:00	Shizan Fang 14:30–15:00
	Ivan Gentil 15:00–15:30	Xiang-Dong Li 15:00–15:30		Alok Goswami 15:00–15:30	Yimin Xiao 15:00–15:30
	Tea Break				
	Fu-Zhou Gong 16:00–16:30	Jinwen Chen 16:00–16:30		Fuqing Gao 16:00–16:30	Shui Feng 16:00–16:30
	Brice Franke 16:30–17:00	Qingyang Guan 16:30–17:00		Xian-Yuan Wu 16:30–17:00	Jie Xiong 16:30–17:00
	Dejun Luo 17:00–17:20	Xin Qi 17:00–17:30		Dapeng Zhan 17:00–17:30	Chunhua Ma 17:00–17:20



## July 14

- 08:30–08:40 Opening (Speech by Mu-Fa Chen)
- 08:40–09:30 Masatoshi Fukushima (Osaka University)  
*Recent progress on boundary theory of Markov processes*
- 09:30–10:00 Tea break, and take picture
- 10:00–10:30 Zhen-Qing Chen (University of Washington)  
*Discrete approximations to reflected Brownian motion*
- 10:30–11:00 Panki Kim (Seoul National University)  
*Intrinsic ultracontractivity for non-symmetric semigroup*
- 11:00–11:30 Jinghai Shao (Beijing Normal University)  
*Modified logarithmic Sobolev inequalities and transportation cost inequalities in Euclidean space*
- 14:30–15:00 Chii-Ruey Hwang (Academia Sinica, Taiwan)  
*Stochastic system: a study of three examples*
- 15:00–15:30 Ivan Gentil (Universite Paris-Dauphine)  
*The Lévy-Fokker-Planck equation:  $\phi$ -entropies and convergence to equilibrium*
- 15:30–16:00 Tea break
- 16:00–16:30 Fu-Zhou Gong (Chinese Academy of Sciences)  
*Essential spectral radius for positive operators on  $L^1$  and  $L^\infty$  spaces*
- 16:30–17:00 Brice Franke (Academia Sinica, Taiwan)  
*About the heat-flow to equilibrium*
- 17:00–17:30 Dejun Luo (Beijing Normal University)  
*Regularity of solutions to differential equations with non-Lipschitz coefficients*

# July 15

08:30–09:00 Shuenn-Jyi Sheu (Academia Sinica, Taiwan)

*Convergence rate to the equilibrium for Brownian motion with divergence free drift*

09:00–09:30 Feng-Yu Wang (Beijing Normal University)

*Coupling method for Harnack inequalities and applications*

09:30–10:00 Tea break

10:00–10:30 Renming Song (University of Illinois at Urbana-Champaign)

*Boundary Harnack principle for subordinate Brownian motions*

10:30–11:00 Christian Leonard (University of Paris X)

*Transport-information inequalities for Markov processes*

11:00–11:20 Xu Zhang (Beijing Normal University and Beijing Univ. of Technology)

*Barta's formula for the principle eigenvalues of Schrödinger operators*

14:30–15:00 Tusheng Zhang (University of Manchester and Nankai University)

*Boundary value problems for elliptic operators with measurable coefficients*

15:00–15:30 Xiang-Dong Li (Universite Paul Sabatier, Toulouse III)

*Riesz transforms, Poincaré inequalities and Hodge theory on complete Riemannian manifolds*

15:30–16:00 Tea break

16:00–16:30 Jinwen Chen (Tsinghua University)

*Nonergodicity of Markov processes*

16:30–17:00 Qingyang Guan (Chinese Academy of Sciences)

*SLE and  $\alpha$ -SLE driven by Lévy processes*

17:00–17:30 Xin Qi (Peking University)

*Functional central limit theorem for spatial birth and death processes*

## July 16

08:30–09:00 Anyue Chen (University of Hong Kong and University of Liverpool)

*Decay properties of Markovian bulk-arriving queues*

09:00–09:30 Jiashan Tang (Nanjing University of Posts and Telecommunications)

*Performance analysis of joining the shortest queue model among a large number of queues*

09:30–10:00 Tea break

10:00–10:30 Xia Chen (University of Tennessee)

*High moment asymptotics for local and intersection local times*

10:30–11:00 Yutao Ma (University Paris X)

*Spectral gap and convex concentration inequalities for birth-death processes*

11:00–11:20 Liang-Hui Xia (Beijing Normal University)

*Spectral gap for quasi-birth-death processes with application to Jackson networks*

## July 17

- 08:30–09:00 Leonid Mytnik (Israel Institute of Technology)  
*Uniqueness for Volterra-type stochastic equation*
- 09:00–09:30 Quansheng Liu (Univ. de Bretagne-Sud., France and Changsha Univ. of Science & Technology)  
*Branching random walks on  $R$  in random environment*
- 09:30–10:00 Tea break, and take picture
- 10:00–10:30 Hao Wang (University of Oregon)  
*A class of interacting superprocesses and their associated SPDEs*
- 10:30–11:00 Xiaowen Zhou (Concordia University)  
*A superprocess with coalescing Brownian spatial motion*
- 11:00–11:20 Hui He (Beijing Normal University)  
*Discontinuous superprocesses with dependent spatial motion*
- 14:30–15:00 Dayue Chen (Peking University)  
*Some open problems of percolation and the contact process on graphs*
- 15:00–15:30 Alok Goswami (Indian Statistical Institute)  
*Random continued fractions*
- 15:30–16:00 Tea break
- 16:00–16:30 Fuqing Gao (Wuhan University)  
*Moderate deviations for Poisson-Dirichlet distribution*
- 16:30–17:00 Xian-Yuan Wu (Capital Normal University)  
*Uniqueness of the critical probability for percolation in the two dimensional Sierpinski carpet lattice*
- 17:00–17:30 Dapeng Zhan (University of California, Berkeley)  
*An Introduction to Stochastic Loewner Evolution (SLE)*

## July 18

- 08:30–09:00 Zengjing Chen (Shandong University)  
*Nonlinear expectation and nonlinear pricing*
- 09:00–09:30 Dong Han (Shanghai Jiao Tong University)  
*Heterogeneous coagulation-fragmentation processes*
- 09:30–10:00 Tea break
- 10:00–10:30 Zongxia Liang (Tsinghua University)  
*Theory of anticipating local time*
- 10:30–11:00 Fubao Xi (Beijing Institute of Technology)  
*On switching diffusions and jump-diffusions*
- 11:00–11:20 Liqun Niu (Beijing Normal University)  
*Some stability results of optimal investment in a simple Lévy*
- 14:30–15:00 Shizan Fang (Universite of Bourgogne)  
*Monge optimal transport maps and Fokker-Planck equations*
- 15:00–15:30 Yimin Xiao (Michigan State University)  
*Sample path properties of Lévy processes*
- 15:30–16:00 Tea break
- 16:00–16:30 Shui Feng (McMaster University)  
*Limiting theorems for Dirichlet processes*
- 16:30–17:00 Jie Xiong (University of Tennessee)  
*A central limit type theorem for particle filter*
- 17:00–17:20 Chunhua Ma (Beijing Normal University)  
*Catalytic discrete state branching models and related limit theorems*



## SPEED OF CONVERGENCE OF CERTAIN MARKOV PROCESSES TO EQUILIBRIUM

Rabi BHATTACHARYA, *The University of Arizona, USA*,  
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**Abstract:** We consider several classes of Markov processes-irreducible as well as non-irreducible, and their approach to equilibrium. A *splitting condition*, generalizing a notion of Dubins and Freedman (1966) , yields speeds of convergence in appropriate metrics for *monotone Markov chains* which are, in general, not irreducible. Examples include iterations of random quadratic maps, non-linear autoregressive models, and the Popp-Wilson algorithm for the Gibbs measure of the Ising model on a finite lattice. Conditions for exponential and polynomial rates of convergence to a steady state are derived for the *Lindley process* which is also monotone, but for which splitting does not occur. *Multi-dimensional diffusions* are another important class of Markov processes for which we explore new criteria for convergence to equilibrium at *polynomial rates*.

## UNIQUENESS AND EXTINCTION PROPERTIES OF INTERACTING BRANCHING COLLISION PROCESSES

Anyue CHEN *University of Liverpool, UK*, E-mail: achen@liverpool.ac.uk  
Junping LI *Central South University, Changsha*

KEY WORDS: Markov branching processes, collision branching processes, interaction, regularity, uniqueness, extinction probability, extinction time

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60J27, Secondary 60J35

**Abstract:** In this talk, we shall address some topics, particularly the uniqueness and extinction properties, of the Interacting Branching Collision Process (IBCP), which consists of two strongly interacting components: an ordinary Markov branching process (MBP) and a collision branching process (CBP). We establish that there is a unique IBCP, and derive necessary and sufficient conditions for it to be non-explosive that are easily to be checked. Explicit expressions are obtained for the extinction probabilities for the regular, critical-explosive and super-explosive cases. The subtle sub-explosive case will also be addressed. The associated expected hitting times are also considered.

### References

- [1] A.Y. Chen, P.K. Pollett, H.J. Zhang, and J.P. Li (2004). The collision branching process. *J. Appl. Prob.*, **41**, 1033-1048.
- [2] A.V. Kalinkin, (2002) Markov branching processes with interaction. *Russian Math. Surveys***57**, 241–304.
- [3] A.V. Kalinkin, (2003) On the extinction probability of a branching process with two kinds of interaction of particles. *Theory Probab. Appl.***46**, 347–352.

## COLLISIONS OF RANDOM WALKS

Dayue CHEN *Peking University, Beijing*, E-mail: dayue@pku.edu.cn

**Abstract:** Run two or more independent random walks on the same graph. Will they meet infinitely often? In this talk we will review what we know, recent progress and open problems.

## LARGE DEVIATIONS FOR NON-ERGODIC MARKOV PROCESSES

Jinwen CHEN *Tsinghua University, Beijing*, E-mail: jchen@math.tsinghua.edu.cn

**Abstract:** Some large deviation and related results for non-ergodic Markov processes, especially for absorbing Markov processes, will be presented.

## BROWNIAN MOTIONS IN A RENORMALIZED POISSONIAN POTENTIALS

Xia CHEN *University of Tennessee, USA*, E-mail: xchen@math.utk.edu

KEY WORDS: Brownian motion, Poisson field

MATHEMATICAL SUBJECT CLASSIFICATION: 60F05

**Abstract:** The model of Brownian motion in Poissonian potential describes a typical trajectory of a Brownian particle surviving from being attracted by the obstacles randomly located in the space (think about the stars in the universe). It also closely related to the study of Anderson models. In the existing literature, the random potential is defined as the convolution between a Poissonian field and a bounded and locally supported function.

According to the Newton's law of universal attraction and some other related laws in physics, the most natural way of constructing the random potential is to define it as the Riesz potential of the Poissonian field. On the other hand, the Riesz potential of the Poissonian field blows up.

In this talk, this problem will be fixed by the way of renormalization. In addition, some asymptotic patterns of our models will be established and more problems will be asked. Part of the talk is based on some collaborative works with Kulic.

## ASYMPTOTIC EXPANSION WITH DOUBLE LAYERS OF FORWARD EQUATIONS OF MARKOV CHAINS

Wan-Chu CHIEN *Institute of Mathematics, Academia Sinica, Taipei*  
 Tzuu-Shuh CHIANG *Institute of Mathematics, Academia Sinica, Taipei*,  
 E-mail: matsch@math.sinica.edu.tw

KEY WORDS: Forward equations, asymptotic expansions

MATHEMATICAL SUBJECT CLASSIFICATION: 60J27, 93E20, 34E05

**Abstract:** For a Markov chain on a finite state space  $\{1, 2, \dots, m\}$  with (possibly inhomogeneous) transition rate matrix

$$Q^\epsilon(t) = 1/\epsilon^2 \cdot Q_2(t) + 1/\epsilon \cdot Q_1(t) + Q_0(t),$$

we consider its Forward equation

$$\partial p / \partial t = p(t) Q^\epsilon(t)$$

$$p^\epsilon(0) = p_0$$

where  $p(t) = (p(X_t = 1), p(X_t = 2), \dots, p(X_t = m))$  is the probability distribution of  $X_t$  and  $p_0$  is any given initial distribution.

We shall show that there exist functions  $f^{(i)}$  (regular part),  $h^{(i)}$  (first layer) and  $g^{(i)}$  (double layer) such that

$$\sup_{0 \leq t \leq T} \left| p^\epsilon - \left( \sum_{i=1}^n \epsilon^i f^{(i)}(t) + \sum_{i=1}^n \epsilon^i h^{(i)}(t/\epsilon) + \sum_{i=1}^n \epsilon^i g^{(i)}(t/\epsilon^2) \right) \right| = O(\epsilon^{n+1}).$$

Moreover, we want to have that  $h^{(i)}(\tau) \leq K \exp(-\tau\gamma)$  and  $g^{(i)}(\tau) \leq K \exp(-\tau\gamma)$  for some positive constants  $\tau$  and  $\gamma$  for each  $h^{(i)}$  and  $g^{(i)}$ .

Previous work [1] considered the asymptotic expansion of Forward equation with two time scales and we generalize their results to three scales using the same matched expansion methods. However, some new phenomena emerge and we shall discuss their probabilistic interpretation.

## References

- [1] George Yin and Qing, Zhang (1997). Continuous-Time Markov Chains and Applications, A Singular Perturbation Approach Springer.

## LIMIT THEOREMS FOR FUNCTIONS OF MARGINAL QUANTILES

G. Jogesh BABU *Department of Statistics, The Pennsylvania State University, USA*

Zhidong BAI *Department of Statistics & Applied Probability, NUS, Singapore*

**Kwok Pui CHOI** *Department of Statistics & Applied Probability, NUS, Singapore,*

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Vasudevan MANGALAM *Department of Mathematics, Universiti Brunei Darussalam, Brunei*

KEY WORDS: Central Limit Theorem, Cramér-Wold device, lost association, quantiles, strong law of large numbers, weak convergence of a process

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60F05, 60F15 Secondary 60F17, 62G30

**Abstract:** Multivariate distributions are explored using the joint distributions of marginal sample quantiles. Limit theory for the mean of a function of order statistics is presented. The results include a multivariate central limit theorem and a strong law of large numbers. A result similar to Bahadur's representation of quantiles is established for the mean of a function of the marginal quantiles. In particular, it is shown that

$$\sqrt{n} \left( \frac{1}{n} \sum_{i=1}^n \phi \left( X_{n:i}^{(1)}, \dots, X_{n:i}^{(d)} \right) - \bar{\gamma} \right) = \frac{1}{\sqrt{n}} \sum_{i=1}^n Z_{n,i} + o_P(1)$$

as  $n \rightarrow \infty$ , where  $\bar{\gamma}$  is a constant and  $Z_{n,i}$  are *i.i.d.* random variables for each  $n$ . This leads to the central limit theorem. Weak convergence to a Gaussian process using equicontinuity of functions is indicated. The results are established under very general conditions. These conditions are shown to be satisfied in many commonly occurring situations.

## SOME RESULTS ON EVOLUTIONARY GAMES

**Yunshyong CHOW** *Institute of Mathematics, Academia Sinica, Taipei,*  
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KEY WORDS: coordination game, prisoner's dilemma game, local interaction,  
long run equilibrium, mutation

MATHEMATICAL SUBJECT CLASSIFICATION: 91A22, 60J20

**Abstract:**  $N$  players sit around a circle to play some game like prisoner's dilemma game or coordination game in which each player has only two strategies to play against his two neighbors. At time period  $n = 1, 2, \dots$ , each player will first find, based on his payoff at time  $n - 1$ , a strategy that would benefit him most in the next play, and then declares his final strategy by allowing to make a mistake independently with a small probability  $\epsilon$ . The setup is like in interacting particle systems and the mutation mechanism is vital as in biological evolutions. The goal is to find  $\lim_{\epsilon \rightarrow 0} \mu_\epsilon$ , where  $\mu_\epsilon$  is the invariant measure as time goes to  $\infty$  with  $\epsilon$  fixed.

For coordination game, there are two Nash equilibria:  $(C, C)$  and  $(D, D)$ . We wish to find out which state, all-C or all-D, will stand out in the long-run.

For prisoner's dilemma game, there is a unique Nash equilibria: (confess, confess). We wish to find out under what conditions on the payoff parameters, one can escape from the all-confess state, so as to be out from the dilemma.

### References

- [1] H.C. Chen & Y. Chow (2009). Evolutionary prisoner's dilemma games with one-dimensional local interaction and imitation, *Advances in Applied Probability*, **41**, 154-176.
- [2] H.C. Chen, Y. Chow & L.C. Wu (2009). Equilibrium selection in coordination games with local interaction and imitation: (I) imitating the average payoff, submitted.
- [3] H.C. Chen, Y. Chow & L.C. Wu (2009). Equilibrium selection in coordination games with local interaction and imitation (II): imitating the total payoffs, submitted.

## A CONTINUUM-TREE-VALUED MARKOV PROCESS

Romain ABRAHAM *University of Orléans, MAPMO; France*  
**Jean-François DELMAS** *University Paris-Est, CERMICS; France,*  
E-mail: delmas@cermics.enpc.fr

KEY WORDS: Continuum random tree, explosion time, pruning, tree-valued Markov process,  
continuous state branching process, exploration process

MATHEMATICAL SUBJECT CLASSIFICATION: 60J25, 60G55, 60J80.

**Abstract:** We present a construction of a Lévy continuum random tree (CRT) (see Duquesne and Le Gall) associated with a super-critical continuous state branching process using the so-called exploration process and a Girsanov theorem. We also extend the pruning procedure developed by Abraham, Delmas and Voisin to this super-critical case. Let  $\psi$  be a critical branching mechanism. We set  $\psi_\theta(\cdot) = \psi(\cdot + \theta) - \psi(\theta)$ . Let  $\Theta = (\theta_\infty, +\infty)$  or  $\Theta = [\theta_\infty, +\infty)$  be the set of values of  $\theta$  for which  $\psi_\theta$  is a conservative branching mechanism. The pruning procedure allows to construct a decreasing Lévy-CRT-valued Markov process  $(\mathcal{T}_\theta, \theta \in \Theta)$ , such that  $\mathcal{T}_\theta$  has branching mechanism  $\psi_\theta$ . It is sub-critical if  $\theta > 0$  and super-critical if  $\theta < 0$ . We then consider the explosion time  $A$  of the CRT: the smallest (negative) time  $\theta$  for which the continuous state branching process (CB) associated with  $\mathcal{T}_\theta$  has finite total mass. We describe the law of  $A$  as well as the distribution of the CRT just after this explosion time. The CRT just after explosion can be seen as a CRT conditioned not to be extinct which is related with an independent intensity related to  $A$ . We also study the evolution of the CRT-valued process after the explosion time. This extends results from Aldous and Pitman on Galton-Watson trees, see also Abraham, Delmas and He for an other approach on Galton-Watson trees. For the particular case of the quadratic branching mechanism, we show that after explosion the total mass of the CB behaves like the inverse of a stable subordinator with index  $1/2$ . This result is related to the size of the tagged fragment for the fragmentation of Aldous' CRT.

## References

- [1] R. ABRAHAM, J.-F. DELMAS, and G. VOISIN (2008). Pruning a Lévy continuum random tree, *Preprint arXiv:0804.1027*.
- [2] R. ABRAHAM, J.-F. DELMAS and H. HE (2010). Pruning Galton-Watson Trees and Tree-valued Markov processes, *Preprint arXiv:1007.0370*
- [3] D. ALDOUS (1990). The continuum random tree II: an overview, In *Proc. Durham Symp. Stochastic Analysis*, pages 23–70. Cambridge univ. press edition.
- [4] D. ALDOUS (1991). The continuum random tree I. *Ann. Probab.*, 19(1):1–28.
- [5] D. ALDOUS (1993). The continuum random tree III, *Ann. Probab.*, 21(1):248–289.
- [6] D. ALDOUS and J. PITMAN (1998). Tree-valued Markov chains derived from Galton-Watson processes, *Ann. Inst. H. Poincaré Probab. Statist.*, 34(5):637–686.
- [7] T. DUQUESNE and J.-F. LE GALL (2002). *Random trees, Lévy processes and spatial branching processes*, volume 281. Astérisque.

## MARTINGALE SOLUTIONS AND MARKOV SELECTION OF STOCHASTIC 3D-NAVIER-STOKES EQUATIONS WITH JUMP

**Zhao DONG** *Chinese Academy of Sciences, Beijing*, E-mail: dzhao@amt.ac.cn  
**Jian Liang ZHAI** *Chinese Academy of Sciences, Beijing*

**Abstract:** We study the existence of Martingale solutions of stochastic 3D Navier-Stokes equations with jump, and the existence of Markov selections for the martingale solutions.

## CONSISTENT MINIMAL DISPLACEMENT OF BRANCHING RANDOM WALKS

Ming FANG *University of Minnesota, USA*, E-mail: fang0086@umn.edu

**Abstract:** Let  $\mathbb{T}$  denote a rooted  $b$ -ary tree and let  $\{S_v\}_{v \in \mathbb{T}}$  denote a branching random walk indexed by the vertices of the tree, where the increments are i.i.d. and possess a logarithmic moment generating function  $\Lambda(\cdot)$ . Let  $m_n$  denote the minimum of the variables  $S_v$  over all vertices at the  $n$ th generation, denoted by  $\mathbb{D}_n$ . Under mild conditions,  $m_n/n$  converges almost surely to a constant, which for convenience may be taken to be 0. With  $\bar{S}_v = \max\{S_w : w \text{ is on the geodesic connecting the root to } v\}$ , define  $L_n = \min_{v \in \mathbb{D}_n} \bar{S}_v$ . We prove that  $L_n/n^{1/3}$  converges almost surely to an explicit constant  $l_0$ . This answers a question of Hu and Shi.

## STOCHASTIC DYNAMICS ASSOCIATED WITH THE POISSON-DIRICHLET DISTRIBUTION AND THE DIRICHLET PROCESS

Shui Feng *McMaster University, Canada*, E-mail: shuifeng@mcmaster.ca

KEY WORDS: Poisson-Dirichlet distribution, Dirichlet process, infinite-dimensional diffusions

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60F10, Secondary 92D10

**Abstract:** Several symmetric diffusion processes are discussed. The corresponding reversible measures include the GEM distribution, the Poisson-Dirichlet distribution, the Dirichlet process, and their two-parameter counterparts.

### References

- [1] J. Bertoin (2008). Two-parameter Poisson–Dirichlet measures and reversible exchangeable fragmentation-coalescence processes. *Combin. Probab. Comput.* **17**, No. 3, 329–337.
- [2] S.N. Ethier and T.G. Kurtz (1981). The infinitely-many-neutral-alleles diffusion model. *Adv. Appl. Probab.* **13**, 429–452.
- [3] S. Feng and W. Sun (2009). Some diffusion processes associated with two parameter Poisson–Dirichlet distribution and Dirichlet process. *Probab. Theory Relat. Fields*, DOI 10.1007/s00440-009-0238-2.
- [4] S. Feng and F.Y. Wang (2007). A class of infinite-dimensional diffusion processes with connection to population genetics. *J. Appl. Prob.* **44**, 938–949.
- [5] L.A. Petrov (2009). Two-parameter family of infinite-dimensional diffusions on the Kingman simplex. *Funct. Anal. Appl.* **43**, No. 4, 279–296.

## LARGE DEVIATIONS UNDER SUBLINEAR EXPECTATIONS

**Fuqing GAO** *School of Mathematics and Statistics, Wuhan University, Wuhan,*  
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KEY WORDS: large deviations, moderate deviations, sublinear expectation

MATHEMATICAL SUBJECT CLASSIFICATION: 60F10, 60J60

**Abstract:** This talk introduces some results on large deviations under sublinear expectations, including large deviations and moderate deviations for independent random variables and large deviations of stochastic differential equations driven by  $G$ -Brownian motion.

### References

- [1] J. Dembo, & O. Zeitouni, (1998). *Large deviations Techniques and Applications*, Springer, New York, 2nd edition.
- [2] F. Q. Gao, (2009). Pathwise properties and homeomorphic flows for stochastic differential equations driven by  $G$ -Brownian motion, *Stochastic Process and their Applications*, **119**, 3356–3382.
- [3] S. G. Peng, (2006).  $G$ -Expectation,  $G$ -Brownian motion and related stochastic calculus of Itô's type, *Proceedings of the 2005 Abel Symposium 2*, Edit. Benth et. al. 541–567.
- [4] S. G. Peng, (2009). Survey on normal distributions, central limit theorem, Brownian motion and the related stochastic calculus under sublinear expectations, *Sci China Ser A*, **52**, 1391–1411.

## CONVERGENCE TO EQUILIBRIUM OF MARKOV PROCESSES AND FUNCTIONAL INEQUALITIES VIA LYAPUNOV CONDITIONS

**Arnaud GUILLIN** *Université Blaise Pascal, France,*  
E-mail: Arnaud.Guillin@math.univ-bpclermont.fr

**Abstract:** we will show here how a simple method, based on Lyapunov condition, allows to prove various type of inequalities (Poincare, logarithmic Sobolev, transportation,...). We will also see how these Lyapunov conditions via coupling method provide useful quantitative results on convergence to equilibrium! for Markov processes. than 1.

## CONSTRAINED CONTINUOUS-TIME MARKOV DECISION PROCESSES IN POLISH SPACES

**Xianping GUO** *Zhongshan University, Guangzhou,* E-mail: mcsgxp@mail.sysu.edu.cn

KEY WORDS: Continuous-time Markov decision process, unbounded transition rates, occupation measure, linear programming, constrained-optimal policy

MATHEMATICAL SUBJECT CLASSIFICATION: 90C40, 60J27

**Abstract:** This talk concerns with continuous-time constrained Markov decision processes (MDPs) in the class of *history-dependent* randomized policies. The transition rates may be *unbounded*, the reward and costs are admitted to be *unbounded from above and from below*, and the state and action spaces are Polish spaces. The optimality criterion to be maximized is expected discounted rewards, and the constraints can be imposed on expected discounted costs. First, we give conditions for the non-explosion of underlying processes and the finiteness of the discounted criteria. Second, by using a technique of occupation measures, we prove that the constrained optimality of the continuous-time MDPs can be transformed to an *equivalent* optimality problem over a class of probability measures. Based on the equivalent optimality problem and a so-called  *$\bar{w}$ -weak convergence* of probability measures developed here, we show the existence of a constrained-optimal policy. Third, by providing a linear programming (LP) formulation of the equivalent optimality problem and its dual problem we show the solvability of constrained-optimal policies, prove that the strong duality condition holds, and further construct a sequence of functions which approximates the constrained optimal reward value. Finally, we use an example to illustrate the main results.

## PRUNING GALTON-WATSON TREES AND TREE-VALUED MARKOV PROCESSES

Romain ABRAHAM *Université d'Orléans, France*

Jean-François DELMAS *Université Paris-Est, France*

Hui HE *Beijing Normal University, Beijing*, E-mail: hehui@bnu.edu.cn

KEY WORDS: Pruning, branching process, Galton-Watson process, random tree, ascension process

MATHEMATICAL SUBJECT CLASSIFICATION: 05C05, 60J80, 60J27

**Abstract:** We present a new pruning procedure on discrete trees by adding marks on the nodes of trees. This procedure allows us to construct and study a tree-valued Markov process  $\{\mathcal{G}(u)\}$  by pruning Galton-Watson trees and an analogous process  $\{\mathcal{G}^*(u)\}$  by pruning a critical or subcritical Galton-Watson tree conditioned to be infinite. Under a mild condition on offspring distributions, we show that the process  $\{\mathcal{G}(u)\}$  run until its ascension time has a representation in terms of  $\{\mathcal{G}^*(u)\}$ . A similar result was obtained by Aldous and Pitman (1998) in the special case of Poisson offspring distributions where they considered uniform pruning of Galton-Watson trees by adding marks on the edges of trees; see also Abraham and Delmas (2010) for continuum-tree-valued models.

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## FEYNMAN-KAC FORMULA FOR HEAT EQUATION DRIVEN BY FRACTIONAL WHITE NOISE

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KEY WORDS: Fractional Brownian field, stochastic partial differential equations, Feynman-Kac formula, Malliavin calculus, smoothness of the density, Hölder continuity

MATHEMATICAL SUBJECT CLASSIFICATION: 60H15

**Abstract:** This is a joint work with David Nualart and Jian Song. In this work we establish a version of the Feynman-Kac formula for the stochastic heat equation with a multiplicative fractional Brownian sheet. We prove the smoothness of the density of the solution, and the Hölder regularity in the space and time variables.

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## ON THE OPTIMAL TRANSITION MATRIX FOR MCMC SAMPLING

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KEY WORDS: Markov chain, Markov chain Monte Carlo, asymptotic variance, average-case analysis, worst-case analysis, rate of convergence, reversibility, non-reversibility

MATHEMATICAL SUBJECT CLASSIFICATION: 60J10, 65C40, 15A42

**Abstract:** Let  $\mathcal{X}$  be a finite space and  $\pi$  be an underlying probability on  $\mathcal{X}$ . For any real-valued function  $f$  defined on  $\mathcal{X}$ , we are interested in calculating the expectation of  $f$  under  $\pi$ . Let  $X_0, X_1, \dots, X_n, \dots$  be a Markov chain generated by some transition matrix  $P$  with invariant distribution  $\pi$ . The time average,  $\frac{1}{n} \sum_{k=0}^{n-1} f(X_k)$ , is a reasonable approximation to the expectation,  $E_\pi[f(X)]$ . Which matrix  $P$  minimizes the asymptotic variance of  $\frac{1}{n} \sum_{k=0}^{n-1} f(X_k)$ ? The answer depends on  $f$ . Rather than a worst-case analysis, we will identify the set of  $P$ 's that minimize the average asymptotic variance, averaged with respect to a uniform distribution on  $f$ .

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## THE GEOMETRIC PROCESS MAINTENANCE MODEL

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KEY WORDS: Geometric process, maintenance problem

MATHEMATICAL SUBJECT CLASSIFICATION: 60H25, 60K20

**Abstract:** In this talk, a geometric process (GP) maintenance model for a repairable system is studied. Lam (1988a, b) introduced the GP which is a simple monotone process. For a deteriorating system, assume that the successive operating times form a decreasing GP, while the consecutive repair times constitute an increasing GP. For an improving system, assume that the successive operating times form an increasing GP, while the consecutive repair times constitute a decreasing GP. A replacement policy  $N$  is applied by which the system will be replaced by a new, identical one following the  $N$ th failure. For the deteriorating system, an optimal replacement policy is determined analytically, and the monotonicity properties of the optimal replacement policy in each of the parameters in the model are then studied. For the improving system, we show that the optimal replacement policy is essentially a policy without replacement.

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## DECAY PROPERTY OF MARKOV BRANCHING PROCESSES WITH IMMIGRATION AND DISASTER

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KEY WORDS: Markov branching processes, state-independent immigration, disaster, decay parameter, invariant measures, invariant vectors, quasi-stationary distributions

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60J27, Secondary 60J35

**Abstract:** We consider decay properties including decay parameter, invariant measures, invariant vectors and quasi-stationary distributions of Markov branching processes with state-independent immigration. In the case that the process is transient, it is important to investigate the decay property of the process. In this paper, the exact value of the decay parameter  $\lambda_Z$  is obtained and expressed explicitly. The invariant measures, invariant vectors, and quasi-distributions are then presented. The generating functions of these invariant measures and quasi-stationary distributions are explicitly presented. The criteria for  $\lambda_Z$ -recurrence and positively  $\lambda_Z$ -recurrence are revealed.

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## MOMENTS, MODERATE AND LARGE DEVIATIONS FOR A BRANCHING PROCESS IN A RANDOM ENVIRONMENT

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KEY WORDS: branching processes, random environment, moments, harmonic moments, large deviations, moderate deviations, central limit theorem.

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 60K37, 60J05, 92D25

**Abstract:** Let  $(Z_n)$  be a supercritical branching process in a random environment  $\xi$ , and  $W = \lim_{n \rightarrow \infty} Z_n / E[Z_n | \xi]$  be the limit of the normalized population size. We show moderate and large deviation principles for the sequence  $\log Z_n$  (with appropriate normalization). In the proof, we calculate the critical value for the existence of harmonic moments of  $W$ , and show an equivalence of the moments of  $Z_n$ .

## THE GREEN-KUBO FORMULA FOR GENERAL MARKOV PROCESSES WITH CONTINUOUS TIME PARAMETER

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KEY WORDS: Green-Kubo formula, Markov processes with continuous time, stochastic evolution equations, interacting diffusions

MATHEMATICAL SUBJECT CLASSIFICATION: 82B35, 60J25, 60H15, 60K35

**Abstract:** For general Markov processes, the Green-Kubo formula is shown to be valid under a mild condition. A class of stochastic evolution equations in a separable Hilbert space and three typical infinite systems of locally interacting diffusions on  $\mathbf{Z}^d$  (irreversible in most cases) are shown to satisfy the Green-Kubo formula, and the Einstein relations for these stochastic evolution equations are shown explicitly as a corollary. In addition, the diffusion coefficient in statistical dynamics and that in probability are shown to be not equivalent sometimes.

## STOCHASTIC DIFFERENTIAL EQUATIONS WITH SOBOLEV COEFFICIENTS

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KEY WORDS: Stochastic differential equation, quasi-invariance, density estimate, Sobolev coefficient

MATHEMATICAL SUBJECT CLASSIFICATION: 60H10, 34F05

**Abstract:** The new approach to Di Perna-Lions flow given by G. Crippa and C. De Lellis, based on maximal functions, has been successfully implemented by X. Zhang to stochastic differential equations. In this work, we will combine X. Zhang's method and the a priori  $L^p$  estimate of density for SDE to study SDE having coefficients in Sobolev spaces.

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## TRANSPORTATION-INFORMATION INEQUALITIES FOR CONTINUUM GIBBS MEASURES

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**Abstract:** The objective of this paper is to establish sharp concentration inequalities for the Glauber dynamics related with continuum or discrete Gibbs measures. At first we establish the optimal transportation-information  $W_1I$ -inequality for the  $M/M/\infty$ -queue associated with the Poisson measure, which improves several previous known results. Under the Dobrushin's uniqueness condition, we obtain some sharp  $W_1I$ -inequalities for the continuum Gibbs measure and for the discrete spin system. Our method is a combination of Lipschitzian spectral gap, the Lyapunov test function approach, and the tensorization technique.

## ALMOST SURE EXPONENTIAL STABILITY OF NUMERICAL METHODS FOR HYBRID STOCHASTIC DIFFERENTIAL EQUATIONS

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KEY WORDS: Brownian motion, backward Euler-Maruyama, Markov chain, almost sure exponential stability

MATHEMATICAL SUBJECT CLASSIFICATION: 65C30, 60H10, 60H35

**Abstract:** Stability analysis of numerical methods for stochastic differential equations (SDEs) has recently received more and more attention. Stability analysis of numerical methods for SDEs is motivated by the question “for what choices of stepsize does the numerical method reproduce the characteristics of the test equation?” In this talk, positive results are derived concerning the long time dynamics of numerical simulations of SDE with Markovian switching (also known as hybrid SDEs). The Euler–Maruyama (EM) discretizations are shown to capture almost sure exponential stability for all sufficiently small timesteps under the local Lipschitz condition and linear growth condition. However, we will show that without the linear growth condition the EM method may not preserve the almost sure exponential stability. We will then show that the backward EM method can capture almost sure exponential stability for a certain class of highly nonlinear hybrid SDEs.

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## SOME RESULTS FOR NAVIER-STOKES EQUATIONS

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**Abstract:** The fundamental difficulty in the study of the Navier-Stokes equations in dimension three is the lack of substantial apriori estimates. Basically only the energy inequality is known. In this talk I'll explain how to use the basic gamma two technique to establish an energy estimate for the vorticity of the velocity vector field which describes an incompressible fluid under periodic boundary condition.

## TWO-SIDED ESTIMATE ON THE GREEN FUNCTIONS OF SUBORDINATE BROWNIAN MOTIONS IN BOUNDED OPEN SETS

**Renming SONG** *University of Illinois, USA,* E-mail: rsong@illinois.edu

**KEY WORDS:** Green functions, Poisson kernels, subordinate Brownian motion, harmonic functions, Harnack inequality, boundary Harnack principle.

**MATHEMATICAL SUBJECT CLASSIFICATION:** Primary 60J45, Secondary 60J25, 60J51.

**Abstract:** Subordinate Brownian motions are obtained from Brownian motion by replacing the time parameter of Brownian motion by an independent subordinator. Subordinate Brownian motions form a large class of Lévy processes, and most of the well-known Lévy processes, such as symmetric stable processes and relativistic stable processes, are subordinate Brownian motions.

In this talk, I will present some recent results in the potential theory of subordinate Brownian motions. In particular, I will discuss sharp two-sided estimates on the Green functions of subordinate Brownian motions in bounded  $C^{1,1}$  open sets.

This talk is based on joint works with Panki Kim and Zoran Vondracek.

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# OPTIMAL TRANSPORTATION, GRADIENT FLOWS AND WASSERSTEIN DIFFUSION

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**Abstract:** We present a brief introduction to recent progress in optimal transportation on manifolds and metric spaces. We recall the characterization of the heat equation on Riemannian manifolds  $M$  as the gradient flow for the relative entropy on the  $L^2$ -Wasserstein space of probability measures  $\mathcal{P}(M)$ , regarded as an infinite dimensional Riemannian manifold. Of particular interest are recent extensions to the heat flow on Finsler spaces, Heisenberg groups and Wiener spaces. Convexity properties of the relative entropy  $Ent(\cdot|m)$  also play an important role in a powerful concept of generalized Ricci curvature bounds for metric measure spaces  $(M, d, m)$ .

Moreover, we give a survey on recent results for the Wasserstein diffusion, a canonical reversible process  $(\mu_t)_{t \geq 0}$  on the Wasserstein space  $\mathcal{P}(\mathbb{R})$ . This includes: particle approximation, logarithmic Sobolev inequality, quasi-invariance of its invariant measure, the so-called entropic measure,  $\mathbb{P}^\beta$  under push forwards  $\mu \mapsto h_*\mu$  by means of smooth diffeomorphisms  $h$  of  $\mathbb{R}$ . We also indicate how to construct the entropic measure on multi-dimensional spaces, formally given as  $\frac{1}{Z} \exp(-\beta \cdot Ent(\cdot|m))\mathbb{P}^0(d\mu)$ .

# BURGERS-KPZ TYPE PDES CHARACTERISING THE PATH-INDEPENDENT PROPERTY OF THE DENSITY OF THE GIRSANOV TRANSFORMATION FOR SDES

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KEY WORDS: SDEs, the Girsanov transformation, nonlinear parabolic PDEs of Burgers-KPZ type, diffusion processes and nonlinear PDEs on differential manifolds

MATHEMATICAL SUBJECT CLASSIFICATION: 60H10, 58J65, 35Q53

**Abstract:** Let  $X_t$  solve the (multidimensional) Itô's SDEs on  $\mathbf{R}^d$

$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dB_t$$

where  $b : [0, \infty) \times \mathbf{R}^d \rightarrow \mathbf{R}^d$  is smooth in its two arguments,  $\sigma : [0, \infty) \times \mathbf{R}^d \rightarrow \mathbf{R}^d \otimes \mathbf{R}^d$  is smooth with  $\sigma(t, x)$  being invertible for all  $(t, x) \in [0, \infty) \times \mathbf{R}^d$ ,  $B_t$  is  $d$ -dimensional Brownian motion. It is shown that, associated to a Girsanov transformation, the stochastic process

$$\int_0^t \langle (\sigma^{-1}b)(s, X_s), dB_s \rangle + \frac{1}{2} \int_0^t |\sigma^{-1}b|^2(s, X_s)ds$$

is a function of the arguments  $t$  and  $X_t$  (i.e., path-independent) if and only if  $b = \sigma\sigma^*\nabla v$  for some scalar function  $v : [0, \infty) \times \mathbf{R}^d \rightarrow \mathbf{R}$  satisfying the time-reversed Burgers-KPZ type equation

$$\frac{\partial}{\partial t} v(t, x) = -\frac{1}{2} [(Tr(\sigma\sigma^*\nabla^2 v))(t, x) + |\sigma^*\nabla v|^2(t, x)].$$

The assertion also holds on a connected complete differential manifold.

## References

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- [2] A. Truman, F.-Y. Wang, J.-L. Wu, W. Yang: A Burgers-KPZ type parabolic equation for the path-independence of the density of the Girsanov transformation. Submitted.

## PHASE TRANSITION ON THE DEGREE SEQUENCE OF A RANDOM GRAPH PROCESS WITH VERTEX COPYING AND DELETION

**Xianyuan WU** *Capital Normal University, Beijing*, E-mail: wuxy@mail.cnu.edu.cn

**Abstract:** This paper focuses on the degree sequence of a random graph process with copying and vertex deletion. A phase transition is revealed as the following: when copying strictly dominates deletion, the model possesses a power law degree sequence; and when deletion strictly dominates copying, it possesses an exponential one; otherwise, the model possesses an intermediate degree distribution which decay as  $e^{-O(\sqrt{k})}$ . Note that, due to copying, the edge number of the model may grow super-linearly and the model may exhibit a power law with any exponent greater than 1.

## WEAKLY COUPLED LEVY TYPE OPERATORS AND SWITCHED LEVY TYPE PROCESSES

**Fubao XI** *Beijing Institute of Technology, Beijing*, E-mail: xifb@bit.edu.cn

**KEY WORDS:** Weakly coupled Lévy type operator, switched Lévy type process, non-explosiveness, Feller property, exponential ergodicity, Harnack inequality

**MATHEMATICAL SUBJECT CLASSIFICATION:** 60J25, 60J27, 60J60, 60J75

**Abstract:** The switched Lévy type process associated with a weakly coupled Lévy type operator is constructed by the Ikeda-Nagasawa-Watanabe piecing together procedure. The non-explosiveness and Feller property are proved for the switched Lévy type process. Based on these results, exponential ergodicity is obtained under the Foster-Lyapunov drift condition. Finally, the Harnack inequality for the switched Lévy type process is proved.

This talk is based on joint work with Zhen-Qing CHEN.

## CRITICAL BROWNIAN SHEET DOES NOT HAVE DOUBLE POINTS

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**KEY WORDS:** 60G60, 60J45, 60G15

**MATHEMATICAL SUBJECT CLASSIFICATION:** Brownian sheet, multiple points, capacity, Hausdorff dimension

**Abstract:** The Brownian sheet is a multiparameter extension of Brownian motion and plays important roles in probability theory and stochastic partial differential equations. Sample functions of the Brownian sheet generate many interesting random fractals and have been studied by many authors.

This talk is concerned with potential theory of the Brownian sheet and its fractal geometric properties. We show that an  $N$ -parameter Brownian sheet in  $\mathbf{R}^d$  has double points if and only if  $2(d - 2N) < d$ . In particular, in the critical case where  $2(d - 2N) = d$ , Brownian sheet does not have double points. This answers an old problem in the folklore of the subject. We also discuss some of the geometric consequences of the mentioned decoupling, and establish a partial result concerning  $k$ -multiple points in the critical case  $k(d - 2N) = d$ .

This talk is based on joint work with Robert Dalang, Davar Khoshnevisan, Eulalia Nualart and Dongsheng Wu.

## References

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## STABILITY IN DISTRIBUTION OF STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

**Chenggui YUAN** *Swansea University, Wales, UK*, E-mail: C.Yuan@Swansea.ac.uk

KEY WORDS: Stochastic partial differential equation, stochastic partial delay differential equation, mild solution, stability in distribution

MATHEMATICAL SUBJECT CLASSIFICATION: 60H15, 60G57

**Abstract:** In this talk, we investigate stochastic partial differential equations. By introducing a suitable metric between the transition probability functions of mild solutions, we derive sufficient conditions for stability in distribution of mild solutions. Consequently, we generalize some existing results of finite dimensional cases to infinite dimensional cases. Finally, some examples are given to demonstrate the applicability of our theory.

## References

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- [2] J. Bao, A. Truman and C. Yuan, Stability in Distribution of Mild Solutions to Stochastic Partial Differential Delay Equations with Jumps, *Proceeding of The Royal Society A* **465** (2009), 2111-2134..

# STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS WITH REFLECTION

**Tusheng ZHANG** *University of Manchester, UK*, E-mail: [tzhang@maths.man.ac.uk](mailto:tzhang@maths.man.ac.uk)

**Abstract:** In this talk I will report some recent progress on stochastic partial differential equations with reflection. In particular, I will discuss the existence, uniqueness, large deviations and invariant measures.

## Participants: (in order of the family name)

**Rabi Bhattacharya:**

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