

# Workshop on Markov Processes and Related Topics

- Period:** August 17 - 19, 2002
- Place:** Room 314, Mathematics Building,  
Beijing Normal University
- Sponsor:** Probability Group, Research Center of Stochastics,  
Beijing Normal University
- Organizers:** Mu-Fa Chen, Feng-Yu Wang, Zeng-Hu Li
- Supporter:** Nation Natural Science Foundation of China,  
No. 10121101

## Schedule

<b>August 17</b>	<b>August 18</b>	<b>August 19</b>
Chairman: Mu-Fa Chen	Chairman: Da-Yue Chen	Chairman: Li-Ming Wu
08:30-08:40 Address		
08:40-09:40 B. Schmitt	08:30-09:30 T.S. Zhang	08:30-09:30 B. Schmuland
10:00-11:00 S. Feng	09:30-10:30 F.Z. Gong	09:30-10:30 X.W. Zhou
11:00-12:00 D.Y. Chen	10:50-11:50 F.Y. Wang	10:50-11:50 Z.Q. Chen
Lunch		
Chairman: Shui Feng	Chairman: Feng-Yu Wang	Chairman: Bernard Schmitt
14:00-15:00 S.Z. Fang	14:00-15:00 E.P. Hsu	14:00-15:00 D. Han
15:00-16:00 X.S. Zhang	15:00-16:00 X.L. Zhao	15:00-16:00 Y.G Lu
16:20-17:10 Y.H. Mao	16:15-17:15 Z.H. Li	16:20-17:20 L.M. Wu
17:10-18:00 Y.Z. Wang	17:15-18:00 Y.H. Zhang	
	Reception Party	

## August 17

- 08:40-09:40 B. Schmitt (University of Bourgogne)  
*Rate of mixing for Gibbs states of dynamical systems*
- 10:00-11:00 S. Feng (McMaster University)  
*Fleming-Viot process: large deviation and quasi-potential*
- 11:00-12:00 D.Y. Chen (Peking University)  
*The metastability of the biased majority vote process*
- 14:00-15:00 S.Z. Fang (University of Bourgogne)  
*Tangent processes and its applications*
- 15:00-16:00 X.S. Zhang (Huadong Normal University)  
*On stochastic order for Diffusion processes*
- 16:20-17:10 Y.H. Mao (Beijing Normal University)  
*Deviation kernels for one-dimensional diffusion processes*
- 17:10-18:00 Y.Z. Wang (Beijing Normal University)  
*Algebraic convergence of Markov chains*

## August 18

- 08:30-09:30 T.S. Zhang (Manchester University)  
*Perturbed reflected Diffusion processes*
- 09:30-10:30 F.Z. Gong (Chinese Academy of Science)  
*Exponential integrability of functions on loop spaces*
- 10:50-11:50 F.Y. Wang (Beijing Normal University)  
*Gradient estimates of Dirichlet heat semigroups and application to isoperimetric inequalities*
- 14:00-15:00 E.P. Hsu (Northwestern University)  
*Brownian motion and Dirichlet problem at infinity*
- 15:00-16:00 X.L. Zhao (Fudan University)  
*Stochastic analysis on  $p$ -adics*
- 16:15-17:15 Z.H. Li (Beijing Normal University)  
*Construction of measure-valued diffusions carried by stochastic flows*
- 17:15-18:00 Y.H. Zhang (Beijing Normal University)  
*Dual variational formulas for the first Dirichlet eigenvalue on half-line*

**August 19**

- 08:30-09:30 B. Schmuland (University of Alberta)  
*A Cocycle proof that reversible Fleming-Viot processes have uniform mutation*
- 09:30-10:30 X.W. Zhou (Concordia University)  
*Coalescing Brownian motion, its dualities and a measure-valued process*
- 10:50-11:50 Z.Q. Chen (University of Washington)  
*Heat kernel estimate for stable-like processes on  $d$ -sets*
- 14:00-15:00 D. Han (Shanghai Jiao Tong University)  
*Gelation of a Reversible Markov Process of Polymerization*
- 15:00-16:00 Y.G. Lu (University of Bari)  
*Quantum Markovian approximation*
- 16:20-17:20 L.M. Wu (Wuhan University)  
*Essential spectral radius for Markov kernel*

# Rate of mixing for Gibbs states of dynamical systems

Bernard Schmitt  
University of Bourgogne

**Abstract:** The  $g$  measures are discrete stationary processes continuously depending on their past, they extend in a natural way the Markov processes. If we assume regularity of the weight  $g$  than the process is strongly mixing and the rate of mixing exponential. We produce a new approach based upon inequalities of Poincare's type for giving constructive estimates of the mixing rate.

# Fleming-Viot process: large deviation and quasi-potential

Shui Feng  
McMaster University

**Abstract:** Fleming-Viot process with neutral mutation is a measure-valued process describing the evolution of genotype frequency in a population under the influence of mutation and resampling. In this talk results will be presented on large deviations and quasi-potential of the Fleming-Viot process. The main difficulty is dealing with the degeneracy of the diffusion coefficient at the boundary. These are joint work with D.A. Dawson, and with Jie Xiong.

# The metastability of the biased majority vote process

Da-Yue Chen  
Peking University

**Abstract:** The reduction method provides an algorithm to compute large deviation estimates of (possibly non reversible) Markov processes with exponential transition rates. It replaces the original graph minimisation equations of Freidlin and Wentzell by more tractable path minimisation problems. We apply this technique to the study of a biased majority vote process generalising the one studied in Chen. We show that this non reversible dynamics has a two well potential with a unique metastable state, and give an upper bound for its relaxation time.

# Tangent processes and its applications

Shi-Zan Fang  
University of Bourgogne

**Abstract:** The Cameron-Martin's quasi-invariance result for translations plays a fundamental role in the theory of stochastic calculus of variations on the Wiener space. When we deal with a non linear situation, the available Cameron-Martin directions are needed to be enlarged: This gives rise to the notion of tangent processes. In this talk, we shall show by examples the role of tangent processes to establish the quasi-invariance results in non linear situations.



# On stochastic order for diffusion processes

Xin-Sheng Zhang  
Huadong Normal University

**Abstract:** Let  $X(t)$  be a diffusion process in  $R^d$ , i.e., let  $X(t)$  satisfy the following stochastic differential equation:

$$dX(t) = b(X(t)) + \sigma(X(t))dW(t),$$

where  $b(x) : R^d \rightarrow R^1$   $\sigma(x)$  is a  $d \times d$  matrix, and  $W(t)$  is a d-dimensional Brownian motion. The usual stochastic order and convex order for two diffusion processes will be discussed. Some sufficient and necessary conditions in terms of  $b(x)$  and  $\sigma(x)$  for two diffusion processes to have usual stochastic order or convex order will be presented.

# Deviation kernels for one-dimensional diffusion processes

Yong-Hua Mao  
Beijing Normal University

**Abstract:** It is proven that for the non-explosive and ergodic diffusion on the half line with the transition probability kernel  $p(t, x, y)$ , the deviation kernel  $d(x, y) = \int_0^\infty (p(t, x, y) - 1)dt$  exists and is finite if and only if  $\int_0^\infty \mathbf{E}^x H_0 \mu(dx) < \infty$ , where  $H_0$  is the hitting time of 0 and  $\mu$  is the speed measure. The explicit formulas are also obtained.

# Algebraic convergence of Markov chains

Ying-Zhe Wang  
Beijing Normal University

**Abstract:** Algebraic convergence in  $L^2$ -sense is studied for general time-continuous, reversible Markov chains with countable state space, and especially for birth-death chains. Some criteria for the convergence are presented. The results are effective since the convergence region can be completely covered, as illustrated by two examples.

# Perturbed reflected diffusion processes

Tu-Sheng Zhang  
Manchester University

**Abstract:** In this talk we will present some recent results on existence and uniqueness of perturbed reflected diffusion processes, which were studied by M.Yor, Le Gall and others before.

# Exponential integrability of functions on loop spaces

Fu-Zhou Gong

Chinese Academy of Science

**Abstract:** Let  $E$  be the loop space over a compact connected Riemannian manifold with a torsion skew symmetric (TSS) connection. Let  $L$  be the Ornstein-Uhlenbeck operator on the loop space  $E$ , and  $f$  be a cylinder function on  $E$ . We first extend the expression of  $LF$ , proved by O.Enchev and D.W. Stroock for the Levi-Civita connection, to a general TSS connection, and then prove that  $f \in \mathcal{D}(L)$  and  $\varepsilon|Lf|^2$  is exponential integrable for some constant  $\varepsilon := \varepsilon(f) > 0$ .

# Gradient estimates of Dirichlet heat semigroups and application to isoperimetric inequalities

Feng-Yu Wang  
Beijing Normal University

**Abstract:** By using probabilistic approaches, some uniform gradient estimates are obtained for Dirichlet heat semigroups on a Riemannian manifold with boundary. As an application, lower bound estimates of isoperimetric constants are presented in terms of functional inequalities.

# Brownian motion and Dirichlet problem at infinity

Elton P. Hsu  
Northwestern University

**Abstract:** We show how to solve the Dirichlet problem at infinity on a Cartan-Hadamard manifold satisfying very generous curvature conditions by estimating the angular oscillation of Brownian motion on such a manifold.

# Stochastic analysis on $p$ -adics

Xue-Lei Zhao  
Fudan University

**Abstract:** Some new results on the stochastic analysis on  $p$ -adics will be introduced.



# Construction of measure-valued diffusions carried by stochastic flows

Zeng-Hu Li

Beijing Normal University

**Abstract:** Let  $\{W(s, y) : s \geq 0, y \in \mathbb{R}\}$  be a two parameter Brownian motion (a time-space white noise). For a smooth and square-integrable function  $h(\cdot)$  on  $\mathbb{R}$  and any  $r \geq 0$  and  $a \in \mathbb{R}$ , given  $x(r, a, r) = a$  the equation

$$x(r, a, t) = a + \int_r^t \int_{\mathbb{R}} h(y - x(r, a, s)) W(ds, dy), \quad t \geq r,$$

has a unique solution  $\{x(r, a, t) : t \geq r\}$ , which defines a isotropic stochastic flow. We consider a stochastic equation for measure-valued process carried by the flow. The equation is driven by a Poisson point process on the space of one-dimensional excursions. A pathwise unique solution of the equation is proved, which gives a measure-valued diffusion process.

# Dual variational formulas for the first Dirichlet eigenvalue on half-line

Yu-Hui Zhang  
Beijing Normal University

**Abstract:** The aim of the paper is to establish two dual variational formulas for the first Dirichlet eigenvalue of second order elliptic operators on half-line. Some explicit bounds of the eigenvalue depending only on the coefficients of the operators are presented. Moreover, the corresponding problems in the discrete case and the higher-order eigenvalues in the continuous case are also studied. This talk is based on a joint work with Mu-Fa Chen and Xiao-Liang Zhao.

# A cocycle proof that reversible Fleming-Viot processes have uniform mutation

Byron Schmuland  
University of Alberta

**Abstract:** Why is the mutation operator associated with a reversible Fleming-Viot process uniform? Our explanation is based on Handa's recent result that reversible distributions must be quasi-invariant under a certain flow, forcing the mutation operator to satisfy a cocycle identity.

# Coalescing Brownian motion, its dualities and a measure-valued process

Xiao-Wen Zhou  
Concordia University

**Abstract:** Coalescing Brownian motion captures the interactions among a system of Brownian motions. It plays a key role in analyzing certain measure-valued processes. Some aspects of coalescing Brownian motion will be discussed in this talk. We will first introduce a characterization theorem. Then we will present two dual relationships involving coalescing Brownian motion. A sketch of the proof will be given. Such a duality can be applied to construct a measure-valued process.

# Heat kernel estimate for stable-like processes on $d$ -sets

Zhen-Qing Chen  
University of Washington

**Abstract:**  $d$ -sets can be regarded as generalizations of fractals. In this talk, we will study stable-like processes on  $d$ -sets, which include reflected stable processes in Euclidean domains as a special case. More precisely, we will establish parabolic Harnack principle and derive sharp two-sided heat kernel estimate for such stable-like processes. Results on the exact Hausdorff dimensions for the graphs of stable-like processes will also be presented. This talk is based on a joint work with Takashi Kumagai.

# Gelation of a reversible Markov process of polymerization

Dong Han

Shanghai Jiao Tong University

**Abstract:** A reversible Markov process as a chemical polymerization model which permits the coagulation and fragmentation reactions is considered. We present a necessary and sufficient condition for the occurrence of a gelation in the process. We show that a gelation transition may or may not occur, depending on the value of the fragmentation strength, and, in case the gelation takes place, a critical value for the occurrence of the gelation and the mass of the gel can be determined by close forms.

# Quantum Markovian approximation

Yun-Gang Lu  
University of Bari

**Abstract:** Markovian approximation are used both in physics and mathematics. The basic idea is to replace a rather complicate evolution of a physical system by a simpler one which is determined by a semigroup. We shall give some useful Markovian approximations and present their intuitive idea, discuss the rigorous mathematical treatment.

# Essential spectral radius for Markov kernel

Li-Ming Wu  
Wuhan University

**Abstract:** Using two new parameters  $\beta_\tau(P)$  and  $\beta_{wc}(P)$  of non-compactness for a positive kernel  $P$  on a Polish space  $E$ , we obtain a new formula of Nussbaum-Gelfand type for the essential spectral radius  $r_{ess}(P)$  on  $b\mathcal{B}$ . Using that formula we show that different known sufficient conditions for geometric ergodicity such as Doeblin's condition, drift condition by means of Lyapunov function, geometric recurrence etc lead to variational formulas of the essential spectral radius. All those can be easily transported on the weighted space  $b_u\mathcal{B}$ . Some related results on  $L^2(\mu)$  are also obtained, especially in the symmetric case. Moreover we prove that for a strongly Feller and topologically transitive Markov kernel, the large deviation principle of Donsker-Varadhan for occupation measures of the associated Markov process holds if and only if the essential spectral radius is zero. The knowledge of  $r_{ess}(P)$  allows us to estimate eigenvalues of  $P$  in  $L^2$  in the symmetric case, and to estimate the geometric convergence rate by means of that in the metric of Wasserstein. Some examples are presented.



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