Extinguishing Behaviors for Continuous-State Nonlinear Branching Processes

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Abstract

A nonnegative continuous-state branching process becomes extinguishing if it converges to 0 (but never hits 0) as time goes to infinity. We consider a class of continuous-state nonlinear processes obtained from spectrally positive stable like Lévy processes by Lamperti type time changes using regularly varying (at 0) rate functions, and obtain several large time asymptotic results on the extinguishing processes. In particular, we show that, depending on whether the stable index for the spectrally positive Lévy process is smaller than or equal to the regularly varying index for the rate function, a phase transition occurs for the convergence of rescaled first passage times of levels approaching 0. We find conditions on convergence in probability and convergence almost surely, respectively. We also obtain integral tests on almost sure long time fluctuation of the running minimum process.

Introduction

Let $Z$ be a spectrally positive Lévy process defined on $(\Omega, F, P)$ with log-Laplace exponent $\psi$, i.e. $Z$ is a stochastic process with stationary independent increments and with no negative jumps, and for $\lambda \geq 0$, $\left\{ e^{\lambda Z_t} : t \geq 0 \right\}$ is a martingale. Let $F_e\Lambda(X, Z_t) = \lambda e^{\lambda X}$, where the Laplace exponent

$$\psi(\lambda) = \lambda \Phi(\lambda) = -\inf_{\lambda > 0} \Phi^{-1}(\lambda)$$

for $\lambda$-finite measure $\nu$ satisfying $\int_1^{\infty} (1-x^2)^\nu(dx) < \infty$. In this paper we only consider process $Z$ that is not a subordinator.

Let $R$ be a positive locally bounded function on $[0, \infty)$ satisfying $\sup_{x > 0} R(x) > 0$ for all $t > 0$. For $t > 0$, let

$$\eta_t = \inf \{ t > 0 : Z_t \leq b \},$$

and $\psi^{-1}$ be the right continuous inverse of $\psi$. Define $X_t = Z_t - \int_0^t r_s \, ds$ for all $t \geq 0$ and $X_0 = 0$. Let $\lambda > 0$ and $\Phi(\lambda) = -\inf_{\lambda > 0} \Phi^{-1}(\lambda)$ be the classical continuous-state branching process that has been well studied. The nonlinear branching mechanism allows more interesting behaviors for the process. In particular, the boundary classification and the associated asymptotic behaviors have been investigated for the continuous-state nonlinear branching processes, the so-called continuous-state nonlinear branching process with branching rate function $\nu$ and branching rate function $\beta$, which was first introduced in Li [3] where the rate function is a power function. Intuitively, it is a generalized continuous-state branching process whose branching rate depends on the current population size, and consequently, it does not have the additive branching property in general. But it remains a Markov process as the time change of a Lévy process. Process $X$ with $X_t = X_0 + \int_0^t \lambda s \, ds$ reduces to the classical continuous-state branching process that has been well studied.

In this paper we continue to investigate the extinguishing behaviors for continuous-state nonlinear branching processes. For any $b > 0$ let

$$T_b = \inf \{ t > 0 : X_t \leq b \},$$

with the convention $\inf \emptyset = \infty$. Let $W(t)$ be a scale function that is the associated spectrally negative Lévy process, i.e.

$$W(t) = X_t - \int_0^t W(s) \, ds - \frac{1}{\lambda} \int_0^t \lambda \Phi(\lambda) \, \, \, d\Lambda,$$

where $\Phi(0) = \sup \{ \lambda \geq 0 : \psi(\lambda) = 0 \}$ and $\Phi(0) = 0$ for $\Phi \leq 0$.

Main results

To investigate the extinguishing behaviors for continuous-state nonlinear branching processes, throughout this paper we always assume that

- the spectrally positive Lévy process $Z$ does not drift to infinity, i.e. $E[Z] \leq 0$.

Given $1 < c < 2$ and $t \geq 0$, we need some of the following assumptions for the main results.

References

