Large deviation principle of occupation measures for non-linear monotone SPDEs

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Outline

What is Large Deviations ?

$\mathcal{X}$ a Polish space, $\mathcal{M}_1(\mathcal{X})$ the probability space. Assume μ_ε weakly converges to the Dirac measure $\delta_p(p \in \mathcal{X})$ in $\mathcal{M}_1(\mathcal{X})$. Then

$$\mu_\varepsilon(A) \longrightarrow 0, \quad \text{as } \varepsilon \to 0, \text{if } p \notin \bar{A}.$$

How to estimate the rate of convergence?
Large deviation principle tells us that

$$\mu_\varepsilon(A) = \exp\{-\inf_{x\in A} I(x)/\lambda(\varepsilon) + o(1/\lambda(\varepsilon))\}$$

for $\inf_{A^o} I = \inf_{\bar{A}} I$, where

- **the rate function:** $I : \mathcal{X} \to [0, +\infty]$ is inf-compact;
- **the speed function**: $\lambda(\varepsilon) > 0, \lim_{\varepsilon\to 0} \lambda(\varepsilon) = 0$

Example 1. Cramér Theorem

$(X_n)_{n\geq 1}$ i.i.d.r.v.'s $(\Omega, \mathcal{F}, \mathbb{P})$, in $\mathbb{R}^d$, law μ. Law of large number:

$$\mathbb{P}\left(\left|\frac{1}{n}\sum_{k=1}^{n} X_k - \mathbb{E}X\right| > \eta\right) \to 0, \quad \text{as } n \to 0.$$

How to estimate this probability? Central limit theorem, Law of iterated logarithm, Bessry-Essen, $\cdots$

Cramér (1938): Assume $\mathbb{E}\exp(\lambda|X|) < \infty$, $\forall \lambda > 0$. Then

$$\mathbb{P}\left(\frac{1}{n}\sum_{k=1}^{n} X_k \in A\right) = \exp\{-n \inf_{A} I + o(n)\}, \quad \text{if } \inf_{A^o} I = \inf_{\bar{A}} I,$$

where $I(x) = \sup_{y\in\mathbb{R}^d}\{\langle x, y\rangle - \Lambda(y)\}$, $\Lambda(y) = \log \mathbb{E}\exp\langle X, y\rangle$.

Example 2. Sanov's theorem

Assume $(X_n)_{n\geq 1}$ i.i.d.r.v.'s on $(\Omega, \mathcal{F}, \mathbb{P})$, valued in a Polish space E, law μ. The empirical measures

$$\mathcal{L}_n := \frac{1}{n}\sum_{k=1}^{n} \delta_{X_k} \in \mathcal{M}_1(E), \quad n \geq 1.$$

Theorem (Sanov (1957))

$\mathbb{P}(\mathcal{L}_n \in \cdot)$ satisfies the LDP on $\mathcal{M}_1(E)$ equipped with the weak convergence topology $\sigma\left(\mathcal{M}_1(E), C_b(E)\right)$, with speed n and with the rate function given by the relative entropy

$$H(\nu \,|\, \mu) = \begin{cases} \int_E \frac{d\nu}{d\mu} \log \frac{d\nu}{d\mu} d\mu, & \text{if } \nu \ll \mu; \\ +\infty, & \text{otherwise}, \end{cases} \tag{1}$$

Sanov's theorem-Continued

Extend Sanov's theorem to stronger topologies.

- τ-topology: $\sigma(\mathcal{M}_1(E), b\mathcal{B}(E))$, Groeneboom et al. (1979)
- Topology of uniform convergence over certain classes of linear functions, Wu (1994), Dembo and Zajic (1997)
- Wasserstein topoloty: W-Wang-Wu (2010) prove that $\mathcal{L}_n$ satisfies the LDP in the Wasserstein metric W_p $(p \in [1, +\infty))$ if and only if

$$\int_E e^{\lambda d^p(x_0, x)} d\mu(x) < +\infty, \forall \lambda > 0, x_0 \in E.$$

Extend those two Theorems to the dependent case?
Such as Markov processes, Martingales, Stationary processes.

Donsker and Varadhan (1970's-1980's) gave the first answers.

Donsker and Varadhan's LDP for Markov processes

Conditions for Lower Bound: there exist a reference measure α, $p(1,x,y) > 0$ a.s., s.t.:

(a1) $p(1,x,dy) = p(1,x,y)\alpha(dy)$.

(a2) $p(1,x,\cdot)$: $E \longrightarrow L^1(\alpha)$ is continuous.

Conditions for Upper Bound:

(b1) $u_n(x) \geq c > 0$, $\forall x, n$.

(b2) $\forall$ compact set $K \subset E$, $\exists$ C_K s.th. $\sup_{x\in K} \sup_n u_n(x) \leq C_K$.

(b3) $-(\frac{\mathcal{L}u_n}{u_n})(x) \equiv V_n(x) \geq -C$, $\forall x, n$.

(b4) $\lim_{n\to\infty} V_n(x) = V(x)$.

(b5) $\{x : V(x) \leq k\}$ is compact, $\forall k < \infty$.

Large deviations of occupation measure for Markov processes

There have been extensive and studies, e.g.,

- Lower bound: de Acosta (1988), Jain (1990), Wu (1993) for essentially irreducible Markov processes.
- Upper bound: Gärtner (1977), Ellis (1985), Stroock (1984) gave a necessary and sufficient condition for good upper bound w.r.t. the weak convergence topology by means of Cramer's method;
  Wu (2000,2004) characters the LDP by means of uniform integrability, essential spectral radius.

Relations between recurrence and LDP

For Markov chain, the following are equivalent (Meyn, Tweedie)

- (Uniform ergodicity) $\sup_x \|P^n(x,\cdot) - \pi\| \longrightarrow 0$.
- (Uniform geometric ergodicity) $\exists r > 0, C < \infty$ s.t.

$$\sup_x \|P^n(x,\cdot) - \pi\| \le Ce^{-nr}.$$

- Aperiodic and $\exists$ *petite* set $K, \exists \lambda > 0$ s.t.

$$\sup_{x \in E} \mathbb{E}_x[e^{\lambda \tau_K}] < \infty.$$

- Doeblin's condition, Lyapunov's condition, $\cdots$

Those conditions DOES NOT imply the Donsker-Varadhan's LDP. See Baxter, Jain, Varadhan (1991) or Bryc, Smolenski (1992). However, Wu (2001) found a characterization of LDP by means of the *hyper-exponential recurrence*.

The hyper-exponential recurrence criterion

$$\tau_K := \inf\{t \geq 0 \ \text{ s.t. } X_t \in K\}, \quad \tau_K^{(1)} := \inf\{t \geq 1 \ \text{ s.t. } X_t \in K\}.$$

Theorem (Wu, 2001)

Assume

- *Strong Feller:* $\exists 0 < T \in \mathbb{T}$ *s.t.* $P_T b\mathcal{B} \subset C_b(E)$
- *Topological irreducible:* $\forall$ *non empty open* U, $x \in E$, $\exists t \in \mathbb{T}, P_t(x, U) > 0$

The following are equivalent:

- *Hyper-exponential recurrence: for some* $\mathcal{A} \subset \mathcal{M}_1(E)$, *any* $\lambda > 0$, $\exists$ *compact set* $K \subset E$, *such that*

$$\sup_{\nu \in \mathcal{A}} \mathbb{E}^\nu e^{\lambda \tau_K} < \infty, \quad \text{and} \quad \sup_{x \in K} \mathbb{E}^x e^{\lambda \tau_K^{(1)}} < \infty. \tag{2}$$

- *LDP:* $\mathbb{P}_\nu(\mathcal{L}_t \in \cdot)$ *satisfies the LDP on* $\mathcal{M}_1(E)$ *w.r.t. the* τ*-topology uniformly for* $\nu \in \mathcal{A}$.

Some Comments about the rate function

Under the Feller assumption:

$$P_t(C_b(E)) \subset C_b(E), \quad \forall t \geq 0,$$

we know that (c.f. Deuschel-Stroock, Wu)

$$J(\nu) = \sup\left\{ -\int \frac{\mathcal{L}u}{u} \mathrm{d}\nu; 1 \leq u \in \mathbb{D}_e(\mathcal{L}) \right\}, \quad \nu \in \mathcal{M}_1(E) \qquad (3)$$

where $\mathbb{D}_e(\mathcal{L})$ is the extended domain of the generator $\mathcal{L}$ of P_t in $C_b(E)$.

One dimensional diffusion process

Consider a $\mathbb{R}$-valued stochastic differential equation

$$dX_t = \sigma(X_t)dB_t + b(X_t)dt,$$

where B is a standard B.M., σ, b are locally Lipchizian, $\sigma > 0$
If for any $\lambda > 0$, there are two constants $c > 0$ and $L > 0$ s.t.

$$\Phi(x) := -c\text{sign}(x)b(x) - \frac{1}{2}c^2\sigma^2(x) \geq \lambda, \quad \forall |x| \geq L,$$

then $\mathcal{L}_t$ satisfies the LDP w.r.t. τ-topology uniformly over compact sets. See Wu (2000).

LDP for SPDE

By using the hyper-exponential recurrence criterion, the LDPs for the occupation measures of SPDEs are studied by

- Gourcy (2007a, 2007b) : Stochastic Navier-Stokes equation, Stochastic Burgers equation
- Jakšić, Nersesyan, Pillet, Shirikyan (2015,2016) Dissipative PDEs with Rough Noise,

By this criterion, we study the LDP for other PSDEs.

LDP for nonlinear monotone SPDEs

Framework:

- **Gelfand triple**

$$V \subset H \equiv H^* \subset V^*,$$

  $(H, \langle \cdot, \cdot \rangle)$: separable Hilbert space;
  V : reflexive Banach space;
  $V \subset H$ is dense, compact.

- $\{W_t\}_{t \geq 0}$ is a cylindrical Q-Wiener process with $Q := I$ on another separable Hilbert space $(U, \langle \cdot, \cdot \rangle_U)$ w.r.t. a complete filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$.

General framework of SPDE

$$\mathrm{d}X(t) = A(X(t))\mathrm{d}t + B(X(t))\mathrm{d}W_t, \qquad (4)$$

(H1) (Hemicontinuity) $\forall v_1, v_2, v \in V$,
$\mathbb{R} \ni s \mapsto {}_{V^*}\langle A(v_1 + sv_2), v\rangle_V$ is continuous.

(H2) (Weak monotonicity) $\exists c_0 \in \mathbb{R}$ s.t. $\forall v_1, v_2 \in V$,

$$2_{V^*}\langle A(v_1)-A(v_2), v_1-v_2\rangle_V + \|B(v_1)-B(v_2)\|^2_{\mathcal{L}_2(U,H)} \le c_0\|v_1-v_2\|^2_H.$$

(H3) (Coercivity) $\exists r > 0,\ c_1, c_3 \in \mathbb{R}, c_2 > 0$ s.t. $\forall v \in V$,

$$2_{V^*}\langle A(v), v\rangle_V + \|B(v)\|^2_{\mathcal{L}_2(U,H)} \le c_1 - c_2\|v\|^{r+1}_V + c_3\|v\|^2_H.$$

(H4) (Boundedness) $\exists c_4 > 0,\ c_5 > 0$ s.t. $\forall v \in V$,

$$\|A(v)\|_{V^*} \le c_4 + c_5\|v\|^r_V.$$

Solution

Definition

A continuous H-valued $\mathcal{F}_t$-adapted process $\{X_t\}_{t\geq 0}$ is called a solution of (4), *if*

$$\mathbb{E}\left[\int_0^t \left(\|X(s)\|_V^{r+1} + \|X(s)\|_H^2\right) \mathrm{d}s\right] < \infty, \quad \forall t > 0, \tag{5}$$

and $\mathbb{P}$*-a.s.*

$$X(t) = X(0) + \int_0^t A(X(s))\mathrm{d}s + \int_0^t B(X(s))\mathrm{d}W(s), \ \ \forall t \geq 0.$$

According to (Krylov-Rozovskii, Prévôt-Röckner), under Conditions (H1)-(H4), for any $X_0 \in L^2(\Omega \to H; \mathcal{F}_0; \mathbb{P})$, (4) admits a unique solution $\{X_t\}_{t\geq 0}$.

Examples

Equation (4) contains the following models

- Stochastic p-Laplace equation ($r > 1$)
- Stochastic generalized porous media equations ($r > 1$)
- Stochastic fast-diffusion equations ($r \in (0, 1]$)

Exponential Ergodicity, Strong Feller, Irreducibility are studied by methods of coupling and Wang's Harnark inequality, see F.Y. Wang, W. Liu, S.Q. Zhang, $\cdots$

Main Results

Recall the occupation measure $\mathcal{L}_t$:

$$\mathcal{L}_t(A) := \frac{1}{t}\int_0^t \delta_{X_s}(A)\mathrm{d}s \quad A \in \mathcal{B}(H). \tag{6}$$

Theorem (W-Xiong-Xu, 2016)

Assume that (H1)-(H4) hold with $r > 1$ and P_t is strong Feller and irreducible in H. Then the family $\mathbb{P}^\nu(\mathcal{L}_T \in \cdot)$ as $T \to +\infty$ satisfies the LDP on $(\mathcal{M}_1(H), \tau)$, with speed T and rate function J, uniformly for any initial measure ν in $\mathcal{M}_1(H)$.

Main Results

Theorem (W-Xiong-Xu, 2016)

Assume that (H1)-(H4) hold with $r \in (0,1]$ and $c_3 < 0$, P_t is strong Feller and irreducible in H, and $C_B := \sup_{u \in H} \|B(u)\|^2_{L^2(U,H)} < \infty$. Let $\lambda_0 \in (0, -\frac{c_3}{2C_B})$ and

$$\mathcal{M}_{\lambda_0,L} := \left\{ \nu \in \mathcal{M}_1(H) : \int_H e^{\lambda_0 \|x\|_H^2} \nu(\mathrm{d}x) \le L \right\}.$$

Then the family $\mathbb{P}^\nu(\mathcal{L}_T \in \cdot)$ as $T \to +\infty$ satisfies the LDP on $(\mathcal{M}_1(H), \tau)$, with speed T and rate function J, uniformly for any initial measure ν in $\mathcal{M}_{\lambda_0,L}$.

Example: Stochastic differential equation

$$\mathrm{d}X_t = b(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t, \quad X_0 = x \in \mathbb{R}^d. \tag{7}$$

There exists $\lambda_i > 0, p > 2$ s.t.

$$\begin{cases} 2\langle x-y, b(x)-b(y)\rangle + \|\sigma(x)-\sigma(y)\|_2^2 \\ \qquad \le \lambda_0 |x-y|^2 (1 \vee \log|x-y|^{-1}), \\ \|\sigma(x)\|_2 \le \lambda_1(1+|x|), \\ \sup_{x\in\mathbb{R}^d} \|\sigma^{-1}(x)\|_2 \le \lambda_2, \\ 2\langle x, b(x)\rangle + \|\sigma(x)\|_2^2 \le -\lambda_3|x|^p + \lambda_4. \end{cases}$$

- Strong Feller, Irreducible, [X.C. Zhang 2009].

Example: Stochastic p-Laplace equation

$$\mathrm{d}X_t = \left[\mathbf{div}(|\nabla X_t|^{p-2}\nabla X_t) - \gamma|X_t|^{q-2}X_t\right]\mathrm{d}t + B\mathrm{d}W_t, \qquad (8)$$

where $\max\{1, 2d/(d+2)\} < p \le 2 < q$ and $\gamma > 0$, B is a Hilbert-Schmidt operator on $L^2(\Lambda)$.

Take

$$V := H_0^{1,p}(\Lambda) \cap L^q(\Lambda), H := L^2(\Lambda).$$

- Conditions (H1)-(H4) with $r > 1$, [Prévôt-Röckner].
- Strong Feller, Irreducibility, [W. Liu, 2009].

Example: Stochastic generalized porous media equations

$$\mathrm{d}X(t) = (L\Psi(X(t)) + \Phi(X(t)))\,\mathrm{d}t + B(X(t))\mathrm{d}W(t).$$

$$L := -(-\Delta)^{\gamma}, \gamma > 0, \Psi(s) := s|s|^{r-1},$$
$$\Phi(s) := cs,\ \ B(x)e_j := b_i(x)j^{-q}e_j,\ \ j \geq 1,$$

$$\begin{cases} |b_i(u) - b_i(v)| \leq b|u - v|, \\ \inf_{u \in H^{\gamma}(D,\mu)} \inf_{i \geq 1} b_i(u) > 0. \end{cases}$$

$r > 1$: **Stochastic generalized porous media equations**

$$L^{r+1}(D,\mu) \subset H^{\gamma}(D,\mu) \subset (L^{r+1}(D,\mu))^*,$$

- Conditions (H1)-(H4) with $r > 1$, [Prévôt-Röckner]
- Strong Feller [F.Y. Wang, 2015], [S.Q. Zhang, 2014]
- Irreducibility [S.Q. Zhang, 2015].

Example: Stochastic fast-diffusion equations

$0 < r < 1$: **Stochastic fast-diffusion equations**:
Take $D = (0, 1) \subset \mathbb{R}$, $1/3 < r < 1$, $\gamma = 1$, $c < 0$.

$$V := L^{r+1}(D, \mu) \bigcap H^1(D, \mu), H := H^1(D, \mu).$$

- (H1)-(H4) hold with $1/3 < r < 1$ and $c_3 < 0$, [Röckner, Ren, Wang, 2007]
- Strong Feller, Irreducibility, [S.Q, Zhang, 2015].

Proof of Theorem ($r > 1$)

By Itô's formula, for $t > s \geq 0$,

$$\begin{aligned}
\mathbb{E}\left[\|X_t\|_H^2\right] \leq & \mathbb{E}\left[\|X_s\|_H^2\right] + c_1(t-s) - c_2 \int_s^t \mathbb{E}\left[\|X_u\|_V^{r+1}\right] \mathrm{d}u \\
& + c_3 \int_s^t \mathbb{E}\left[\|X_u\|_H^2\right] \mathrm{d}u \\
\leq & \mathbb{E}\left[\|X_s\|_H^2\right] + C_1(t-s) - C_2 \int_s^t \mathbb{E}\left[\|X_u\|_H^{r+1}\right] \mathrm{d}u
\end{aligned}$$

$$\Longrightarrow \quad \sup_{t\geq 1} \mathbb{E}\left[\|X_t\|_H^2\right] \leq C$$

$$\Longrightarrow \quad \int_1^2 \mathbb{E}\left[\|X_u\|_V^{r+1}\right] \mathrm{d}u \leq C$$

$$\Longrightarrow \quad \exists t \in [1,2] \; s.t. \quad \mathbb{E}\left[\|X_t\|_V^{r+1}\right] \leq C,$$

where C is independent of X_0.

By the Markov property of X, there exists a sequence of times $\{t_n; n \geq 1\}$ such that $t_n \in [2n-1, 2n]$ and

$$\mathbb{E}\left[\|X_{t_n}\|_V\right] \leq C. \qquad (9)$$

For any $M > 0$,

$$K := \{x \in V : \|x\|_V \leq M\} \text{ is compact in } H.$$

Based on (9), we have

$$\sup_{\nu \in \mathcal{M}_1(H)} \mathbb{E}^{\nu}[e^{\lambda \tau_K}] < \infty.$$

Proof of Theorem ($r \in (0,1]$, $c_3 < 0$)

Inspirited by [Gourcy, 07]

- For any $0 < \lambda_0 < -c_3/(2C_B)$, $x \in H$,

$$\mathbb{E}^x\left[\exp\left(\frac{\lambda_0 c_2}{2}\int_0^t \|X_s\|_V^r \mathrm{d}s\right)\right] \le e^{\lambda_0 c_1 t}\cdot e^{\lambda_0\|x\|_H^2}.$$

- $K := \{x \in V : \|x\|_V \le M\}$ is compact in H.

$$\begin{aligned}
&\mathbb{P}^\nu(\tau_K^{(1)} > n) \le \mathbb{P}^\nu\left(\mathcal{L}_n(K^c) \ge 1 - \frac{1}{n}\right)\\
\le&\mathbb{P}^\nu\left(\mathcal{L}_n(\|x\|_V^r) \ge M^r\left(1-\frac{1}{n}\right)\right)\\
\le&\exp\left\{-\frac{n\lambda_0 c_2 M^r}{2}\left(1-\frac{1}{n}\right)\right\}\mathbb{E}^\nu\left[\exp\left(\frac{\lambda_0 c_2}{2}\int_0^n \|X_s\|_V^r \mathrm{d}s\right)\right]\\
\le&\exp\{-n\lambda_0 C M^r\}\,\mathbb{E}^\nu\left[e^{\lambda_0\|x\|_H^2}\right].
\end{aligned}$$

Stochastic Ginzburg-Landau equation driven by α-stable noises

$$\begin{cases} \mathrm{d}X_t - \Delta X_t \mathrm{d}t = (X_t - X_t^3)\mathrm{d}t + \mathrm{d}L_t, \\ X_0 = x_0, \end{cases} \tag{10}$$

where $L_t = \sum_{k \in \mathbb{Z}_*} \beta_k l_k(t) e_k$ is an α-stable process on H with $\{l_k(t)\}_{k \in \mathbb{Z}_*}$ being i.i.d. 1-dimensional symmetric α-stable process sequence with $\alpha > 1$.
Assume that there exist some $C_1, C_2 > 0$ so that

$$C_1 \gamma_k^{-\beta} \leq |\beta_k| \leq C_2 \gamma_k^{-\beta}, \quad \beta > \frac{1}{2} + \frac{1}{2\alpha}.$$

The equation

We say that a predictable H-valued stochastic process $X = (X_t^x)$ is a mild solution to Eq. (10) if, for any $t \geq 0, x \in H$,

$$X_t^x = e^{-At}x + \int_0^t e^{-A(t-s)}(X_s^x - (X_s^x)^3)\mathrm{d}s + \int_0^t e^{-A(t-s)}\mathrm{d}L_s.$$

Theorem (Xu (SPA 2013) , W-Xiong-Xu (Bernoulli 2017))

If $\alpha \in (3/2, 2)$ *and* $\frac{1}{2} + \frac{1}{2\alpha} < \beta < \frac{3}{2} - \frac{1}{\alpha}$, *the followings hold*

1. *Eq.* (10) *admits a unique mild solution.*
2. $(X_t^x)_{t\geq 0, x\in H}$ *is strong Feller and irreducible in* H.
3. $(X_t^x)_{t\geq 0, x\in H}$ *is exponential ergodic.*

Theorem (W.-Xiong-Xu, 2016)

Assume that $\alpha \in (3/2, 2)$ and $\frac{1}{2} + \frac{1}{2\alpha} < \beta < \frac{3}{2} - \frac{1}{\alpha}$. Then $\mathbb{P}_\nu(\mathcal{L}_T \in \cdot)$ as $T \to +\infty$ satisfies the LDP w.r.t. the τ-topology, with speed T and rate function J, uniformly for any initial measure ν in $\mathcal{M}_1(H)$.

Difficulty: Due to the discontinuity and the lack of second moment, Itô's formula can't be used directly like the Wiener case. However, the strong coercive nonlinearity $x - x^3$ paves way to produce the hyper-exponential recurrence.

Lemma

For all $T > 0$, $\delta \in (0, 1/2), p \in (0, \alpha/4)$,

$$\mathbb{E}_x \left[\|X_T\|_{H_\delta}^p \right] \le C_{T,\delta,p},$$

where $C_{T,\delta,p}$ does not depend on $X_0 = x$.

SDPE driven by subordinate BM

Let S_t be $\alpha/2$-stable process on $\mathbb{R}$ independent with the cylindrical B.M. W, Q_β be a Hilbert-Schmit operator

$$\mathrm{d}X - \Delta X \mathrm{d}t = (X - X^3)\mathrm{d}t + Q_\beta \mathrm{d}W_{S_t}, \tag{11}$$

Theorem (W.-Xu, 2017)

For $\alpha \in (1,2)$, the Markov process X is strong Feller and irreducible in H, $\mathbb{P}_\nu(\mathcal{L}_T \in \cdot)$ as $T \to +\infty$ satisfies the LDP w.r.t. the τ-topology, with speed T and rate function J, uniformly for any initial measure ν in $\mathcal{M}_1(H)$.

Difficulty: Non-independence of the components in the noise brings some new challenges, especially in the proof of irreducibility.

Critical lemma in the proof of irreducibility.

Lemma

For any $T > 0, p > 0$, *the random variable* $(\{W_{S_t}\}_{0\le t\le T}, W_{S_T})$ *has a full support in* $L^p([0,T];\mathbb{V}) \times \mathbb{V}$. *More precisely, for any* $\phi \in L^p([0,T];\mathbb{V}), a \in \mathbb{V}, \varepsilon > 0$,

$$\mathbb{P}\left(\int_0^T \|W_{S_t} - \phi_t\|_{\mathbb{V}}^p \mathrm{d}t + \|W_{S_T} - a\|_{\mathbb{V}} < \varepsilon\right) > 0. \tag{12}$$

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Thanks for your attention!