The $M^X/M/c$ queue with catastrophes and state-dependent control at idle time

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Background

- Generalized $M/M/1$ with disasters:

- $BMAP/SM/1$ queue with Markovian input of disasters. Dudin A and Karolik A. (2001, Performance Evaluation)

-The $M/M/1$ queue with mass exodus and mass arrives when empty. Chen A Y, Renshaw E. (1997, J Appl Prob.)
- BD-processes with catastrophes. Di Crescenzo A, Giorno V, Nobile A G and Ricciardi L M. (SPL, 2008)

Background

- $M_t/M_t/N$ Queue with Catastrophes. Zeifman A and Korotysheva A. (2012, Stochastic Models)

-The $M/M/c$ Queue With Mass Exodus and Mass Arrivals When Empty. Zhang L, Li J. (2015, JAP.)

Background

The model considered in this talk.

- The $M^X/M/c$ queue with catastrophes and control at idle time

- State space: $\mathbf{E} = \{0, 1, 2, \cdots\}$

-Definition of Q-matrix $Q = (q_{ij};\ i, j \in \mathbf{E})$:

$$Q = Q^* + Q_s + Q_d \tag{1.1}$$

where $Q^* = (q^*_{ij};\ i, j \in \mathbf{E})$, $Q_s = (q^{(s)}_{ij};\ i, j \in \mathbf{E})$ and $Q_d = (q^{(d)}_{ij};\ i, j \in \mathbf{E})$ are all conservative Q-matrices which are given as follows

Background

$$q^*_{ij} = \begin{cases} \min(i,c)b_0, & \text{if } i \geq 1,\ j = i-1, \\ b_1 - [\min(i,c) - 1]b_0, & \text{if } i \geq 1,\ j = i, \\ b_{j-i+1}, & \text{if } i \geq 1,\ j \geq i+1, \\ 0, & \text{otherwise}, \end{cases} \tag{1.2}$$

$$q^{(s)}_{ij} = \begin{cases} -h, & \text{if } i = 0,\ j = 0, \\ h_j, & \text{if } i = 0,\ j \geq 1, \\ 0, & \text{otherwise}, \end{cases} \tag{1.3}$$

$$q^{(d)}_{ij} = \begin{cases} \beta, & \text{if } i \geq 1,\ j = 0, \\ -\beta, & \text{if } i \geq 1,\ j = i, \\ 0, & \text{otherwise}, \end{cases} \tag{1.4}$$

Background

Here $\beta \geq 0,\ h_j \geq 0\ (j \geq 1)$ and $b_j \geq 0\ (j \neq 1)$ with

$$0 \leq h := \sum_{j=1}^{\infty} h_j < \infty \ \ and \ \ 0 < -b_1 = \sum_{j\neq 1} b_j < \infty.$$

Since Q is conservative and bounded, there exists a unique Q-process, i.e., Feller minimal Q-process. We call this process a modified $M^X/M/c$ queueing process and denoted by $\{X_t;\ t \geq 0\}$.

In order to avoid trivial cases, we assume that $b_0 > 0$ and $\sum_{j=2}^{\infty} b_j > 0$.

Background

- Special Cases:

- $c = 1, \beta = 0$: Chen A Y, Renshaw E. (2004, A Appl. Prob.)

- $c = 1, \beta = 0, h_j = b_{j+1}$: Covers $M^X/M/1$ queue.

- $c = 1, b_j = 0\ (j \geq 3)$: Covers Chen A Y, Renshaw E. (1997, J Appl. Prob.)

-$h_1 = b_2$ and $h_j = b_{j+1} = 0\ (j \geq 2)$: Covers Zhang and Li (2015, J Appl. Prob.)

Background

- Problems:

(1) Case $\beta = 0$: for the $M^X/M/c$ with resurrection, recurrence and ergodicity criteria? Busy period? Equilibrium distribution? Equilibrium size?

(2) Case $\beta > 0$: How about the first catastrophe?

Background

Some notations. Denote $\tilde{Q} = Q^* + Q_s$.

● Q-process $\{X_t\}$: $(p_{ij}(t); i, j \in \mathbf{Z})$ for the Q-function and $(r_{ij}(\lambda); i, j \in \mathbf{Z})$ for the Q-resolvent.

● $\tilde{Q}$-process $\{\tilde{X}_t\}$: $(\tilde{p}_{ij}(t); i, j \in \mathbf{Z})$ for the Q-function and $(\tilde{r}_{ij}(\lambda); i, j \in \mathbf{Z})$ for the Q-resolvent.

● Q^*-process $\{X^*_t\}$: $(p^*_{ij}(t); i, j \in \mathbf{Z})$ for the Q-function and $(\phi^*_{ij}(\lambda); i, j \in \mathbf{Z})$ for the Q-resolvent.

Preliminary

Define the generating functions

$$B(s) = \sum_{j=0}^{\infty} b_j s^j \tag{2.1}$$

$$B_i(s) = B(s) + (i-1)b_0(1-s), \quad i = 1, 2, \cdots, c. \tag{2.2}$$

$$H(s) = \sum_{j=1}^{\infty} h_j s^j \tag{2.3}$$

All the above functions are well defined on $[-1, 1]$.

Preliminary

Lemma 1. The equation $B_c(s) = 0$ has a smallest root u on $[0, 1]$ with $u = 1$ if $B_c'(1) \leq 0$ and $u < 1$ if $B_c'(1) > 0$. More specifically,
(i) if $B_c'(1) \leq 0$, then $B_c(s) > 0,\ s \in [0, 1)$.
(ii) if $B_c'(1) > 0$, then $B_c(s) = 0$ has exactly two roots, u and 1, in $[0, 1]$ such that $B_c(s) > 0,\ 0 \leq s < u$ and $B_c(s) < 0,\ u < s < 1$.

We also define for any $\lambda > 0$,

$$U_\lambda(s) := B_c(s) - \lambda s. \tag{2.4}$$

It is clear that for any fixed $\lambda > 0$, the equation $U_\lambda(s) = 0$ has exactly one root $u(\lambda)$ on $[0, 1]$ and $0 < u(\lambda) < 1$.

Preliminary

Lemma 2. For $u(\cdot)$ as defined above.
(i) $u(\lambda) \in C^{\infty}(0,\infty)$;
(ii) $u(\lambda)$ is a decreasing function of $\lambda > 0$;
(iii) $u(\lambda) \downarrow 0$ and $\lambda u(\lambda) \to cb_0$ as $\lambda \to \infty$;
(iv) when $\lambda \to 0^+$,

$$u(\lambda) \uparrow u \begin{cases} = 1 & \text{if } B_c'(1) \leq 0, \\ < 1 & \text{if } B_c'(1) > 0, \end{cases} \tag{2.5}$$

where u is the smallest root of $B_c(s) = 0$ on $[0,1]$;
(v) for any positive integer k,

$$\lim_{\lambda \to 0^+} \frac{1 - u(\lambda)^k}{\lambda} = \begin{cases} \infty & \text{if } B_c'(1) \geq 0, \\ \frac{k}{-B_c'(1)} & \text{if } B_c'(1) < 0. \end{cases} \tag{2.6}$$

Conclusions

- The stopped $M^X/M/c$ process

Theorem 1. For any $i \geq 0$, $(\phi^*_{ij}(\lambda);\ 0 \leq j \leq c-1)$ is the unique solution of the equations

$$\begin{cases} -\lambda\phi^*_{i0}(\lambda) - \sum_{k=1}^{c-1} u(\lambda)^{k-1}[B_c(u(\lambda)) - B_k(u(\lambda))]\phi^*_{ik}(\lambda) = -u(\lambda)^i, \\ -\lambda\phi^*_{i0}(\lambda) + b_0\phi^*_{i1}(\lambda) = -\delta_{i0}, \\ (b_1 - \lambda)\phi^*_{i1}(\lambda) + 2b_0\phi^*_{i2}(\lambda) = -\delta_{i1}, \\ \qquad \cdots \\ \sum_{k=1}^{j-1} \phi^*_{ik}(\lambda)b_{j-k+1} + [b_1 - (j-1)b_0 - \lambda]\phi^*_{ij}(\lambda) + (j+1)b_0\phi^*_{ij+1}(\lambda) = -\delta_{ij}, \\ \qquad \cdots \\ \sum_{k=1}^{c-3} \phi^*_{ik}(\lambda)b_{c-k-1} + [b_1 - (c-3)b_0 - \lambda]\phi^*_{ic-2}(\lambda) + (c-1)b_0\phi^*_{ic-1}(\lambda) = -\delta_{ic-2} \end{cases}$$

where $u(\lambda)(\lambda > 0)$ is the unique root of $U_\lambda(s) = 0$ on $[0, 1]$.

Conclusions

Furthermore, by Theorem 1 and Kolmogorov forward equation, we can obtain all the resolvent $(\phi^*_{ij}(\lambda);\ i,j \in \mathbf{E})$ of the transition probability $(p^*_{ij}(t); i,j \in \mathbf{E})$.

Now denote

$$\tau^*_0 = \inf\{t > 0;\ X^*_t = 0\}$$

$$e^*_k = P(\tau^*_0 < \infty | X^*_0 = k) \quad (k \geq 1)$$

$$m^*_i(k) = \int_0^\infty p^*_{ki}(t)dt \ (i \geq 1)$$

Conclusions

Theorem 2. For the Q^*-process $\{X_t^*;\ t \geq 0\}$, we have
(i) If $B_c'(1) \leq 0$, then $e_k^* = 1,\ (k \geq 1)$;
(ii) if $B_c'(1) > 0$, then $e_k^* = b_0 m_1^*(k)$ $(k \geq 1)$ and for fixed $k \geq 1$, $(m_i^*(k);\ 1 \leq i \leq c-1)$ is the unique solution of the equations

$$\begin{cases} b_0 m_1^*(k) = u^k - \sum_{i=1}^{c-1} m_i^*(k) u^{i-1}(c-i) b_0 (1-u), \\ b_1 m_1^*(k) + 2 b_0 m_2^*(k) = -\delta_{k1}, \\ \quad \cdots \\ \sum_{i=1}^{j-1} b_{j-i+1} m_i^*(k) + [b_1 - (j-1) b_0] m_j^*(k) + (j+1) b_0 m_{j+1}^*(k) = -\delta_{kj}, \\ \quad \cdots \\ \sum_{i=1}^{c-3} b_{c-i-1} m_i^*(k) + [b_1 - (c-3) b_0] m_{c-2}^*(k) + (c-1) b_0 m_{c-1}^*(k) = -\delta_{kc-2}, \end{cases}$$

where u is the smallest root of $B_c(s) = 0$ on $[0, 1]$.

Conclusions

Moreover, all the $(m_i^*(k);\ k \geq 1,\ i \geq 1)$ can be obtained.

(iii) The mean extinction time is

$$E(\tau_0^*|X_0^* = k) = \begin{cases} -\frac{1}{B_c'(1)}[k + \sum_{i=1}^{c-1} m_i^*(k)(c-i)b_0] & \text{if } B_c'(1) < 0, \\ \infty & \text{if } B_c'(1) \geq 0, \end{cases}$$

where $(m_i^*(k);\ 1 \leq i \leq c-1)$ is given by (ii).

Conclusions

- The $M^X/M/c$ process with resurrection ($\tilde{Q}$-process)

By Theorem 2, we have

Corollary 1. For the $M^X/M/c$ queue with resurrection, the mean busy period is

$$B = \begin{cases} -\frac{1}{hB_c'(1)} \sum_{k=1}^{\infty} h_k[k + \sum_{i=1}^{c-1} m_i^*(k)(c-i)b_0], & if \ B_c'(1) < 0 \\ \infty, & if \ B_c'(1) \geq 0. \end{cases}$$

Let

$$\tilde{P}(t) = (\tilde{p}_{ij}(t);\ i, j \geq 0)$$

$$\tilde{R}(\lambda) = (\tilde{r}_{ij}(\lambda);\ i, j \geq 0)$$

be the $\tilde{Q}$-function and $\tilde{Q}$-resolvent, respectively.

Conclusions

Similar to the proof of Theorem 3.1 in Chen and Renshaw [3], using the resolvent decomposition theorem, we have

Theorem 3. For $\tilde{R}(\lambda) = (\tilde{r}_{ij}(\lambda);\ i, j \geq 0)$, we have

$$\tilde{r}_{00}(\lambda) = \left[\lambda + \lambda \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} h_i \phi^*_{ij}(\lambda)\right]^{-1}, \tag{3.1}$$

$$\tilde{r}_{i0}(\lambda) = \tilde{r}_{00}(\lambda) b_0 \phi^*_{i1}(\lambda) \quad (i \geq 1), \tag{3.2}$$

$$\tilde{r}_{0j}(\lambda) = \tilde{r}_{00}(\lambda) \sum_{i=1}^{\infty} h_i \phi^*_{ij}(\lambda) \quad (j \geq 1), \tag{3.3}$$

$$\tilde{r}_{ij}(\lambda) = \phi^*_{ij}(\lambda) + \tilde{r}_{i0}(\lambda) \sum_{k=1}^{\infty} h_k \phi^*_{kj}(\lambda) \quad (i, j \geq 1), \tag{3.4}$$

where $\Phi^*(\lambda) = (\phi^*_{ij}(\lambda);\ i, j \geq 0)$ is the Q^*-resolvent.

Conclusions

Remark 1. $(\tilde{r}_{ij}(\lambda);\ i,j \geq 0)$ can be obtained. Indeed, by Theorem 1, we can get $(\phi^*_{ij}(\lambda);\ i,j \geq 1)$. Then by (3.1) and (3.2), we can get $\tilde{r}_{00}(\lambda)$ and $\tilde{r}_{i0}(\lambda)$, hence, by (3.3), we can obtain $\tilde{r}_{0j}(\lambda)$ $(j \geq 1)$. Finally, $(\tilde{r}_{ij}(\lambda);\ i,j \geq 1)$ can be obtained by (3.4).

Conclusions

Theorem 4. For the modified $M^X/M/c$ queueing process with q-matrix $\tilde{Q}$, we have
(i) the process is recurrent iff $B_c'(1) \leq 0$,
(ii) the process is positive recurrent iff $B_c'(1) < 0$ and $H'(1) < \infty$.

Conclusions

Theorem 5. Suppose theat $B_c'(1) < 0$ and $H'(1) < \infty$. The equilibrium generating function $\tilde{\Pi}(s)$ takes the form

$$\tilde{\Pi}(s) = \tilde{\pi}_0\Big[1 + \frac{s(h - H(s))}{B_c(s)}\Big] + \frac{1}{B_c(s)} \sum_{k=1}^{c-1} \tilde{\pi}_k s^k (c-k) b_0 (1-s),$$

where $\tilde{\pi}_0 = -B_c'(1)\Big[-B_c'(1) + H'(1) + \sum_{k=1}^{c-1} r_k (c-k) b_0\Big]^{-1}$, $\tilde{\pi}_k = \tilde{\pi}_0 r_k \quad (k \geq 1)$ and $\{r_k;\ k = 1, \cdots, c+1\}$ are determined by

$$\begin{cases} b_0 r_1 = h, \\ b_1 r_1 + 2 b_0 r_2 = -h_1, \\ \quad \cdots \\ \sum_{i=1}^{c-1} b_{c-i+1} r_i + [b_1 - (c-1) b_0] r_c + c b_0 r_{c+1} = -h_c, \\ \sum_{i=1}^{j-1} b_{j-i+1} r_i + [b_1 - (c-1) b_0] r_j + c b_0 r_{j+1} = -h_j \quad (j \geq c+1). \end{cases}$$

Conclusions

- The $M^X/M/c$ process with resurrection and catastrophes (Q-process)

By Chen and Renshaw [6], we have

Theorem 6. For $R(\lambda) = (r_{ij}(\lambda);\ i, j \geq 0)$, we have

$$r_{00}(\lambda) = \Big[\lambda + \lambda \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} h_i \phi^*_{ij}(\lambda + \beta)\Big]^{-1}, \quad (3.5)$$

$$r_{i0}(\lambda) = r_{00}(\lambda)\Big[b_0 \phi^*_{i1}(\lambda + \beta) + \beta \sum_{k=1}^{\infty} \phi^*_{ik}(\lambda + \beta)\Big] \quad (i \geq 1), \quad (3.6)$$

$$r_{0j}(\lambda) = r_{00}(\lambda) \sum_{i=1}^{\infty} h_i \phi^*_{ij}(\lambda + \beta) \quad (j \geq 1), \quad (3.7)$$

$$r_{ij}(\lambda) = \phi^*_{ij}(\lambda + \beta) + r_{i0}(\lambda) \sum_{k=1}^{\infty} h_k \phi^*_{kj}(\lambda + \beta) \quad (i, j \geq 1), \quad (3.8)$$

where $\Phi^*(\lambda) = (\phi^*_{ij}(\lambda);\ i, j \geq 0)$ is the Q^*-resolvent.

Conclusions

Lemma 3. For $(\phi_{ij}^*(\lambda);\ i,j\geq 0)$, denote $L_j(\lambda)=\sum_{i=1}^{\infty}h_i\phi_{ij}^*(\lambda)$ $(j\geq 0)$. Then $(L_j(\lambda);\ 0\leq j\leq c-1)$ is the unique solution of the following linear equations

$$\begin{cases} -\lambda L_0(\lambda)-\sum_{k=1}^{c-1}u(\lambda)^{k-1}[B_c(u(\lambda))-B_k(u(\lambda))]L_k(\lambda)=-H(u(\lambda)),\\ -\lambda L_0(\lambda)+b_0L_1(\lambda)=0,\\ (b_1-\lambda)L_1(\lambda)+2b_0L_2(\lambda)=-h_1,\\ \qquad\cdots\\ \sum_{k=1}^{j-1}L_k(\lambda)b_{j-k+1}+[b_1-(j-1)b_0-\lambda]L_j(\lambda)+(j+1)b_0L_{j+1}(\lambda)=-h_j,\\ \qquad\cdots\\ \sum_{k=1}^{c-3}L_k(\lambda)b_{c-k-1}+[b_1-(c-3)b_0-\lambda]L_{c-2}(\lambda)+(c-1)b_0L_{c-1}(\lambda)=-h_{c-2}, \end{cases}$$

where $u(\lambda)(\lambda>0)$ is the unique root of $U_\lambda(s)=0$ on $[0,1]$. Moreover, all the $(L_j(\lambda);\ j\geq 0)$ can be obtained.

Conclusions

Theorem 7. If $h, \beta > 0$, then the Q-process is always positive recurrent. The equilibrium distribution of the Q-process is given by

$$\pi_0 = \beta\Big[\beta + H(1) - H(u(\beta)) + \sum_{k=1}^{c-1} L_j(\beta)u(\beta)^{k-1}(c-k)b_0(1-u(\beta))\Big]^{-1},$$

$$\pi_j = \pi_0 L_j(\beta) \quad (j \geq 1)$$

and $(L_j(\beta);\ j \geq 1)$ is given by Lemma 3.

Conclusions

Theorem 8. The equilibrium queue size, N, has expectation

$$E(N) = \pi_0 \frac{(H(u(\beta)) - h - H'(1))(-\beta) - (H(u(\beta)) - h)(B_c'(1) - \beta)}{\beta^2} + \frac{\sum_{k=1}^{c-1} \pi_k (c-k) b_0 [\beta + u(\beta)^{k-1}(1 - u(\beta))B_c'(1)]}{\beta^2}$$

and the equilibrium waiting queue size, L_w, has expectation

$$E(L_w) = \pi_0 \Big[\frac{(H(u(\beta)) - h - H'(1))(-\beta) - (H(u(\beta)) - h)(B_c'(1) - \beta)}{\beta^2} + c\Big] + \frac{\sum_{k=1}^{c-1} \pi_k (c-k)[b_0(\beta + u(\beta)^{k-1}(1 - u(\beta))B_c'(1)) + \beta^2]}{\beta^2} - c,$$

where $(\pi_k;\ 0 \le k \le c-1)$ is given in Theorem 7.

Conclusions

Now, we consider the first effective catastrophe occurrence time of the Q-process $\{X_t;\ t \geq 0\}$.

The following lemma reveals that $(p_{ij}(t))$ can be expressed in terms of $(\tilde{p}_{ij}(t))$.

Lemma 4. For all $j, n \in \mathbf{E},\ t > 0$, we have

$$p_{jn}(t) = e^{-\beta t}\tilde{p}_{jn}(t) + \beta \int_0^t e^{-\beta s}\tilde{p}_{0n}(s)ds, \qquad (3.9)$$

or

$$r_{jn}(\lambda) = \tilde{r}_{jn}(\lambda+\beta) + \frac{\beta}{\lambda}\tilde{r}_{0n}(\lambda+\beta), \quad \lambda > 0. \qquad (3.10)$$

Conclusions

Let C_{j0} denote the first occurrence time of an effective catastrophe when $X_0 = j$ $(j \in \mathbf{E})$, and let $d_{j0}(t)$ $(t > 0)$ be the density of C_{j0}.

In order to study C_{j0}, we consider a modified process $\{M_t;\ t \geq 0\}$. Its behavior is identical to that of $\{X_t;\ t \geq 0\}$ before catastrophe, the only difference is that the effect of a catastrophe from state $n > 0$ makes a jump from n to the absorbing state -1.

Let $H(t) = (h_{jn}(t),\ j, n \in \mathbf{S})$ and $\eta(\lambda) = (\eta_{jn}(\lambda),\ j, n \in \mathbf{S})$ be the Q_M-function and Q_M-resolvent, respectively.

By the relation of $\{X_t; t \geq 0\}$ and $\{M_t; t \geq 0\}$, we have

$$P(C_{j0} > t) = \sum_{n=0}^{+\infty} h_{jn}(t) = 1 - h_{j,-1}(t), \quad j \in \mathbf{E}. \tag{3.11}$$

Conclusions

Now we need to consider $(\eta_{jn}(\lambda);\ j,n\in\mathbf{S})$.

Theorem 9. For all $j\in\mathbf{E}$ and $\lambda>0$, we have

$$\eta_{j,-1}(\lambda)=\frac{\beta}{\lambda+\beta}\left[\frac{1}{\lambda}-\frac{\tilde{r}_{j0}(\lambda+\beta)}{1-\beta\tilde{r}_{00}(\lambda+\beta)}\right], \tag{3.12}$$

$$\eta_{jn}(\lambda)=\tilde{r}_{jn}(\lambda+\beta)+\beta\tilde{r}_{0n}(\lambda+\beta)\frac{\tilde{r}_{j0}(\lambda+\beta)}{1-\beta\tilde{r}_{00}(\lambda+\beta)},\quad n\geq 0, \tag{3.13}$$

where $(\tilde{r}_{jn}(\lambda+\beta);\ j,n\in\mathbf{E})$ is given by Remark 1.

Conclusions

Sketch of proof.

Step 1. Comparing the Kolmogorov forward equation for $(\eta_{0j}(\lambda); j \geq 0)$ and $(r_{0j}(\lambda); j \geq 0)$, we can get

$$\eta_{0n}(\lambda) = \frac{\lambda}{\lambda + \beta - \lambda\beta r_{00}(\lambda)} \cdot r_{0n}(\lambda), \quad n \geq 0. \tag{3.14}$$

By Lemma 4, we obtain (3.13) with $j = 0$.

Step 2. Comparing the Kolmogorov forward equation for $(\eta_{jn}(\lambda); n \geq 0)$ and $(r_{jn}(\lambda); n \geq 0)$ with $j \geq 1$, we can get

$$\eta_{jn}(\lambda) = D_j(\lambda) r_{jn}(\lambda) + F_j(\lambda) r_{0n}(\lambda), \quad j \geq 1, \quad n \geq 0. \tag{3.15}$$

where

$$D_j(\lambda) = 1, \quad F_j(\lambda) = \frac{\beta[\lambda r_{j0}(\lambda) - 1]}{\lambda + \beta - \lambda\beta r_{00}(\lambda)}, \quad j \geq 1. \tag{3.16}$$

Conclusions

By Lemma 4, we obtain (3.13) with $j \geq 1$.
Finally, noting $h'_{j,-1}(t) = \beta(1 - h_{j,-1}(t) - h_{j,0}(t))$ and (3.13) with $n = 0$ we get (3.12).

Conclusions

Corollary 2. For all $j \in \mathbf{E}$, we have

$$E(C_{j0}) = \frac{1}{\beta} + \frac{\tilde{r}_{j0}(\beta)}{1-\beta\tilde{r}_{00}(\beta)},$$

$$D(C_{j0}) = \frac{1}{\beta^2}\{1 - \frac{\beta^2\tilde{r}_{j0}^2(\beta)}{[1-\beta\tilde{r}_{00}(\beta)]^2} - \frac{2\beta^2}{1-\beta\tilde{r}_{00}(\beta)}\tilde{r}'_{j0}(\beta) - \frac{2\beta^3\tilde{r}_{j0}(\beta)}{[1-\beta\tilde{r}_{00}(\beta)]^2}\tilde{r}'_{00}(\beta)\},$$

where $\tilde{r}_{j0}(\beta)$ $(j \in \mathbf{E})$ are given by Remark 1.

Conclusions

Corollary 3. (i) If $B_c'(1) < 0$ and $H'(1) < \infty$, then

$$\lim_{\beta\downarrow 0} \beta E(C_{j0}) = \frac{-B_c'(1) + H'(1) + \sum_{k=1}^{c-1} r_k(c-k)b_0}{H'(1) + \sum_{k=1}^{c-1} r_k(c-k)b_0}, \quad j \in \mathbf{E}; \tag{3.17}$$

(ii) if $B_c'(1) > 0$, then

$$\lim_{\beta\downarrow 0}[E(C_{j0}) - \frac{1}{\beta}] = \frac{b_0 m_1^*(j)}{h - H(u) + \sum_{k=1}^{c-1} u^k (c-k) b_0 (1-u) r_k}, \quad j \geq 0, \tag{3.18}$$

where $m_1^*(0) = b_0^{-1}$, $(m_1^*(j);\ j \geq 1)$ and $(r_k;\ 1 \leq k \leq c-1)$ are given by Theorems 2 and 5, respectively.

Conclusions

Corollary 4.

$$\lim_{\beta\to+\infty} E(C_{00}) = \frac{1}{h},$$

$$\lim_{\beta\to+\infty} \beta E(C_{j0}) = \begin{cases} 1+\frac{b_0}{h}, & j=1, \\ 1, & j\geq 2. \end{cases}$$

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