ASYMPTOTIC PROPERTIES OF REGIME-SWITCHING STOCHASTIC DAMPING HAMILTONIAN SYSTEMS

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Abstract: This work focuses on a class of stochastic Hamiltonian systems with both damping and continuous-state-dependent switching. First, for a special Markovian switching case, the existence of a globally weak solution is constructed by making using of the martingale approach. Next, for the general state-dependent switching case, the existence of a globally weak solution is established by virtue of the Radon-Nikodym derivative method. Then, strong Feller property is proved by the killing technique and the Radon-Nikodym derivative method with a truncation argument. Based on these results, exponential ergodicity is obtained under the Foster-Lyapunov drift condition. Finally, some examples are presented for illustration.

References

[1] M.F. Chen (2004), From Markov Chains to Non-Equilibrium Particle Systems, Second Edition, World Scientific, Singapore.

[2] Z.Q. Chen, Z. Zhao (1996), Potential theory for elliptic systems, *Ann. Probab.*, 24, 293–319.

[3] D.A. Dawson, X. Zheng (1991), Law of large numbers and central limit theorem for unbounded jump mean-field models, *Adv. Appl. Math.*, 12, 293–326.

[4] S. Feng, X. Zheng (1992), Solutions of a class of nonlinear master equations, *Stoch. Proc. Appl.*, 43, 65–84.

[5] N. Ikeda, S. Watanabe (1981), Stochastic Differential Equations and Diffusion Processes, North-Holland, Amsterdam.

[6] S.P. Meyn, R.L. Tweedie (1993), Markov Chains and Stochastic Stability, Springer-Verlag, Berlin.

[7] S.P. Meyn, R.L. Tweedie (1993), Stability of Markovian processes III: Foster-Lyapunov criteria for continuous-time processes, *Adv. Appl. Probab.*, 25, 518–548.

[8] K. Narita (1991), The Smoluchowski-Kramers approximation for stochastic Liénard equation with mean-field, *Adv. Appl. Probab.*, 23, 303–316.

[9] E. Priola, F.Y. Wang (2006), Gradient estimates for diffusion semigroups with singular coefficients, *J. Funct. Anal.*, 236, 244–264.

[10] J.H. Shao, F.B. Xi (2013), Strong ergodicity of the regime-switching diffusion processes, *Stoch. Proc. Appl.*, 123, 3903–3928.

[11] L.M. Wu (2001), Large and moderate deviations and exponential convergence for stochastic damping Hamiltonian systems, *Stoch. Proc. Appl.*, 91, 205–238.

[12] F.B. Xi, G. Yin (2015), Stochastic Lienard equations with state-dependent switching, *Acta Math. Appl. Sinica, English Series*, 31, 893–908.

[13] F.B. Xi, C. Zhu (2017), On Feller and strong Feller properties and exponential ergodicity of regime-switching jump diffusion processes with countable regimes, *SIAM J. Control Optim.*, 55, 1789–1818.