HARMONIC MOMENTS AND LOWER LARGE DEVIATIONS FOR A SUPERCRITICAL BRANCHING PROCESS IN A RANDOM ENVIRONMENT

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Abstract: Let $(Z_n)_{n\geq 0}$ be a supercritical branching process in an independent and identically distributed random environment $\xi = (\xi_n)_{n\geq 0}$. We study the asymptotic behavior of the harmonic moments $\mathbb{E}[Z_n^{-r}|Z_0 = k]$ of order $r > 0$ as $n \to \infty$, when the process starts with k initial individuals. We exhibit a phase transition with the critical value $r_k > 0$ determined by the equation $\mathbb{E}p_1^k(\xi_0) = \mathbb{E}m_0^{-r_k}$, where $m_0 = \sum_{j=0}^{\infty} jp_j(\xi_0)$, $(p_j(\xi_0))_{j\geq 0}$ being the offspring distribution given the environment ξ_0. Contrary to the constant environment case (the Galton-Watson case), this critical value is different from that for the existence of the harmonic moments of $W = \lim_{n\to\infty} Z_n/\mathbb{E}(Z_n|\xi)$. The aforementioned phase transition is linked to that for the rate function of the lower large deviation for Z_n. As an application, we obtain a lower large deviation result for Z_n under weaker conditions than in previous works and give a new expression of the rate function, and improve an earlier result about the convergence rate in the central limit theorem for $W - W_n$.