ASYMPTOTIC BEHAVIOUR OF EXPONENTIAL FUNCTIONALS OF LÉVY PROCESSES WITH APPLICATIONS TO RANDOM PROCESSES IN RANDOM ENVIRONMENT

Sandra Palau *Centro de Investigación en Matemáticas, México*
Juan Carlos Pardo *Centro de Investigación en Matemáticas, México*
Charline SMADI *Laboratoire d'Ingénierie des Systèmes Complexes, Irstea, France*, E-mail: charline.smadi@irstea.fr

KEY WORDS: Lévy processes, exponential functional, continuous state branching processes in random environment, explosion and extinction probabilities, logistic process, diffusions in random environment.

MATHEMATICAL SUBJECT CLASSIFICATION: 60G17, 60G51, 60G80.

Abstract: Let $\xi = (\xi_t, t \geq 0)$ be a real-valued Lévy process and define its associated exponential functional as follows

$$I_t(\xi) := \int_0^t \exp\{-\xi_s\} ds, \qquad t \geq 0.$$

Motivated by applications to stochastic processes in random environment, we study the asymptotic behaviour of

$$\mathbb{E}\Big[F\big(I_t(\xi)\big)\Big] \qquad \text{as} \qquad t \to \infty,$$

where $F = (F(x), x \geq 0)$ is a function with polynomial decay at infinity and which is non increasing for large x. In particular, under some exponential moment conditions on ξ, we find five different regimes that depend on the shape of the Laplace exponent of ξ. Our proof relies on a discretization of the exponential functional $I_t(\xi)$ and is closely related to the behaviour of functionals of semi-direct products of random variables.

We apply our results to three questions associated to stochastic processes in random environment. We first consider the asymptotic behaviour of extinction and explosion for self-similar continuous state branching processes in a Lévy random environment. Secondly, we focus on the asymptotic behaviour of the mean population size in a model with competition or logistic growth which is affected by a Lévy random environment and finally, we study the tail behaviour of the maximum of a diffusion in a Lévy random environment.