THE NON-DEGENERATE LIMIT FOR SUPERCRITICAL SUPERPROCESSES

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Abstract: Let $X = \{X_t, t \geq 0; \mathbf{P}_\mu\}$ be a supercritical superprocesses in a space E, such that $X_0 = \mu$ with $\mu \in \mathcal{M}_F(E)$. Suppose ϕ_0 is the first eigenfunction corresponding to the first eigenvalue $\lambda_0(> 0)$ of the mean semigroup of X. Then $M_t := e^{-\lambda_0 t}\langle \phi_0, X_t \rangle$ is a martingale which has limit M_∞. It was proved in Liu-Ren-Song (2009) that M_∞ is nondegenerate iff a moment condition, called the $L \log L$ condition, is satisfied. In this talk I will discuss the case that the $L \log L$ condition is not satisfied. We prove that there is a non-trivial family of backward iterates γ_t and a non-degenerate random variable W such that for any $\mu \in \mathcal{M}_F(E)$,

$$\lim_{t \to \infty} \gamma_t \langle \phi_0, X_t \rangle = W, \qquad \text{a.s.-}\mathbf{P}_\mu.$$

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