ASYMPTOTIC PROPERTIES OF MAXIMUM LIKELIHOOD ESTIMATOR FOR THE GROWTH RATE OF AN α-STABLE CIR PROCESS

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Abstract: We consider an α-stable Cox–Ingersoll–Ross (CIR) process

$$\mathrm{d}Y_t = (a - bY_t)\,\mathrm{d}t + \sigma\sqrt{Y_t}\,\mathrm{d}W_t + \delta\sqrt[\alpha]{Y_{t-}}\,\mathrm{d}L_t, \qquad t \in [0,\infty),$$

with a deterministic initial value $y_0 \in [0,\infty)$, where $a \in [0,\infty)$, $b \in (-\infty,\infty)$, $\sigma, \delta \in (0,\infty)$, $\alpha \in (1,2)$, $(W_t)_{t\in[0,\infty)}$ is a 1-dimensional standard Wiener process, and $(L_t)_{t\in[0,\infty)}$ is a spectrally positive strictly α-stable Lévy process with characteristic function

$$\mathrm{E}(\mathrm{e}^{\mathrm{i}\theta L_1}) = \exp\left\{\int_0^\infty (\mathrm{e}^{\mathrm{i}\theta z} - 1 - \mathrm{i}\theta z) C_\alpha z^{-1-\alpha}\,\mathrm{d}z\right\}, \qquad \theta \in (-\infty,\infty),$$

where $C_\alpha := (\alpha\Gamma(-\alpha))^{-1}$ and Γ denotes the Gamma function. We suppose that $(W_t)_{t\in[0,\infty)}$ and $(L_t)_{t\in[0,\infty)}$ are independent.

Supposing that a, σ, δ and L are known, we study asymptotic properties of the maximum likelihood estimator (MLE) for the growth rate b of the model based on continuous time observations $(Y_t)_{t\in[0,T]}$ as $T \to \infty$. If $a \in (0,\infty)$ or $y_0 \in (0,\infty)$, then for each $T \in (0,\infty)$, we show that there exists a unique MLE $\widehat{b}_T$ of b almost surely having the form

$$\widehat{b}_T = -\frac{Y_T - y_0 - aT - \delta\int_0^T \sqrt[\alpha]{Y_{u-}}\,\mathrm{d}L_u}{\int_0^T Y_s\,\mathrm{d}s},$$

provided that $\int_0^T Y_s\,\mathrm{d}s$ is positive (which holds almost surely).

We distinguish three cases: subcritical, critical and supercritical cases according to $b > 0$, $b = 0$ and $b < 0$. In all cases, somewhat surprisingly, we prove strong consistency of the MLE in question. In the subcritical case we prove asymptotic normality. In the supercritical case asymptotic mixed normality is shown, although the limiting mixed normal (but non-normal) distribution is given in a complicated way. In the critical case the description of the asymptotic behaviour of the MLE is still an open question, even the scaling factor is unclear.

A similar analysis on the asymptotic behaviour of the MLE of the growth rate for another jump-type CIR process will be presented by Mátyás Barczy.