DECOMPOSITION OF LEVY TREES ALONG THEIR DIAMETER

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Abstract: We consider the diameter of Levy trees that are random compact metric spaces obtained as the scaling limits of Galton-Watson trees. Levy trees have been introduced by Le Gall and Le Jan (1998) and they generalise Aldous' Continuum Random Tree (1991) that corresponds to the Brownian case. We first characterize the law of the diameter of Levy trees and we prove that it is realized by a unique pair of points. We prove that the law of Levy trees conditioned to have a fixed diameter r is obtained by glueing at their respective roots two independent size-biased Levy trees conditioned to have height r/2 and then by uniformly re-rooting the resulting tree; we also describe by a Poisson point measure the law of the subtrees that are grafted on the diameter. This decomposition relies on a similar one for Levy trees along the geodesic realizing their height that has been obtained by Abraham and Delmas (2009). The law obtained by glueing two trees with height r/2 can be viewed as a natural law for unrooted Levy trees: this can be justified thanks to a limit theorem for unrooted unlabelled planar trees conditioned by their total height that has been obtained in the recent preprint Wang (2016). As an application of this decomposition of Levy trees according to their diameter, we characterize the joint law of the height and the diameter of stable Levy trees conditioned by their total mass; we also provide asymptotic expansions of the law of the height and of the diameter of such normalized stable trees, which generalizes the identity due to Szekeres (1983) in the Brownian case. Note that Szekeres' result has been proved in a simple way and extended by Wang (2015).