ASYMPTOTIC PROPERTIES OF MAXIMUM LIKELIHOOD ESTIMATOR FOR THE GROWTH RATE FOR A JUMP-TYPE CIR PROCESS

Mátyás BARCZY *University of Debrecen, Hungary*, E-mail: barczy.matyas@inf.unideb.hu
Mohamed Ben Alaya *Université Paris 13, France*
Ahmed Kebaier *Université Paris 13, France*
Gyula Pap *University of Szeged, Hungary*

Key words: jump-type Cox–Ingersoll–Ross process, subordinator, maximum likelihood estimator

Mathematical Subject Classification: 60H10, 91G70, 60F05, 62F12

Abstract: We consider a jump-type Cox–Ingersoll–Ross (CIR) process

$$\mathrm{d}Y_t = (a - bY_t)\,\mathrm{d}t + \sigma\sqrt{Y_t}\,\mathrm{d}W_t + \mathrm{d}J_t, \qquad t \in [0,\infty),$$

with a deterministic initial value $y_0 \in [0,\infty)$, where $a \in [0,\infty)$, $b \in (-\infty,\infty)$, $\sigma \in (0,\infty)$, $(W_t)_{t\in[0,\infty)}$ is a 1-dimensional standard Wiener process, and $(J_t)_{t\in[0,\infty)}$ is an independent subordinator (an increasing Lévy process) with zero drift and with Lévy measure m concentrating on $(0,\infty)$ such that $\int_0^\infty z\,m(\mathrm{d}z) \in [0,\infty)$, that is,

$$\mathrm{E}\left(\mathrm{e}^{uJ_t}\right) = \exp\left\{t\int_0^\infty (\mathrm{e}^{uz}-1)\,m(\mathrm{d}z)\right\}, \qquad t \in [0,\infty), \qquad u \in (-\infty,0].$$

We study asymptotic properties of the maximum likelihood estimator (MLE) for the growth rate b of the model based on continuous time observations $(Y_t)_{t\in[0,T]}$ as $T\to\infty$. We distinguish three cases: subcritical, critical and supercritical cases according to $b>0$, $b=0$ and $b<0$. In the subcritical case we prove weak consistency and asymptotic normality, and, under the additional moment assumption $\int_0^1 z\log\left(\frac{1}{z}\right)m(\mathrm{d}z)<\infty$, strong consistency as well. In the supercritical case, we prove strong consistency and mixed normal (but non-normal) asymptotic behavior, while in the critical case, weak consistency and non-standard asymptotic behavior are described. We specialize our results to so-called basic affine jump-diffusions as well. Concerning the asymptotic behavior of the MLE in the supercritical case, we derive a stochastic representation of the limiting mixed normal distribution, where the almost sure limit of an appropriately scaled jump-type supercritical CIR process comes into play. This is a new phenomena, compared to the critical case, where a diffusion-type critical CIR process plays a role.

The full presentation of our results can be found in [1]. A similar analysis on the asymptotic behaviour of the MLE of the growth rate for a so-called α-stable CIR process will be presented by Gyula Pap.

References

[1] M. Barczy, M. Ben Alaya, A. Kebaier, G. Pap (2016). Asymptotic properties of maximum likelihood estimator for the growth rate for a jump-type CIR process based on continuous time observations. *ArXiv* **1609.05865**, 35 pages.