Stochastic Heat Equations	Intermittency	Dissipation	Thank

Dissipation in parabolic SPDEs

Shang-Yuan Shiu joint work with D. Khoshnevisan, K.W. Kim and C. Mueller

Department of Mathematics National Central University

12th Workshop on Markov Processes and Related Topics July 14, 2016

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Consider the following PAM:

$$\frac{\partial u_t(x)}{\partial t} = \bigtriangleup u_t(x) + \xi(x)u_t(x) \ (t \in \mathbf{R}^+, x \in \mathbf{Z}^d),$$
with initial $u_0(x) = 1$ for all $x \in \mathbf{Z}^d$,

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 Parabolic Anderson Model [PAM] - from the point view of particle system

Consider the following PAM:

$$\frac{\partial u_t(x)}{\partial t} = \triangle u_t(x) + \xi(x)u_t(x) \ (t \in \mathbf{R}^+, x \in \mathbf{Z}^d),$$

with initial $u_0(x) = 1$ for all $x \in \mathbb{Z}^d$, where $\xi(x)$ is a function and \triangle is the discrete Laplacian:

$$\bigtriangleup g(x) := \frac{1}{2d} \sum_{|y-x|=1} [g(y) - g(x)].$$

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• On each site $x \in \mathbf{Z}^d$, we put a particle.

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• On each site $x \in \mathbf{Z}^d$, we put a particle.

 At site x, the particle splits into two identical particles with rate ξ⁺(x) or dies with rate ξ⁻(x).

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- Let n(t, x) denote the number of particles at time t and at site x.

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- At site x, the particle splits into two identical particles with rate ξ⁺(x) or dies with rate ξ⁻(x).
- Let n(t, x) denote the number of particles at time t and at site x.
- Define $u_t(x) = E[n(t,x)]$, then $u_t(x)$ solves the PAM.

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Stochastic Heat Equations [SHE]

Consider the following SHE:

$$\frac{\partial}{\partial t}u_t(x) = \triangle u_t(x) + \sigma(u_t(x))\frac{\partial^2}{\partial t\partial x}W(t,x), x \in D \subseteq \mathbf{R}$$

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- $\sigma: \mathbf{R} \to \mathbf{R}$ is globally Lipschitz continuous.
- $\{W(t,x), t > 0, x \in [-1,1]\}$ is a Brownian sheet $(\partial_t \partial_x W(t,x))$ is a centered generalized Gaussian random field with covariance

 $Cov(\partial_t \partial_x W(t,x), \partial_t \partial_x W(s,y)) = \delta_0(t-s)\delta(x-y).)$

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$$Cov(\partial_t \partial_x W(t,x), \partial_t \partial_x W(s,y)) = \delta_0(t-s)\delta(x-y).)$$

• u_0 is nonrandom, measurable and $u_0(x)$ is uniformly bounded away from zero and infinity.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Mild solution			

$$u_t(x) = \int_D p_t(x,y)u_0(y)dy + \int_{(0,t]\times D} p_{t-s}(x,y)\sigma(u_s(y))W(dsdy).$$

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• SHE on the whole domain **R**, $p_t(x, y)$ is the transition density of Brownian motion.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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 SHE with Dirichlet boundary condition on [a, b] (u_t(a) = u_t(b) = 0, t > 0),

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- SHE on the whole domain **R**, $p_t(x, y)$ is the transition density of Brownian motion.
- SHE with Dirichlet boundary condition on [a, b] (u_t(a) = u_t(b) = 0, t > 0), p_t(x, y) is the transition density of Killed Brownian motion.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Mild solution			

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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Mild solution			

$$u_t(x) = \int_D p_t(x, y) u_0(y) dy + \int_{(0,t] \times D} p_{t-s}(x, y) \sigma(u_s(y)) W(dsdy).$$

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- SHE with Dirichlet boundary condition on [a, b] (ut(a) = ut(b) = 0, t > 0), pt(x, y) is the transition density of Killed Brownian motion.
- SHE with Neumann boundary condition on [a, b] (∂_xu_t(a) = ∂_xu_t(b) = 0, t > 0), p_t(x, y) is the transition density of reflected Brownian motion.

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Mild solution			

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 SHE with periodic boundary condition on [a, b] (u_t(a) = u_t(b), ∂_xu_t(a) = ∂_xu_t(b), t > 0),

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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- SHE with periodic boundary condition on [a, b] (ut(a) = ut(b), ∂xut(a) = ∂xut(b), t > 0), pt(x, y) is the transition density of the Brownian motion on the torus.

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Markov Property			

$$u_t(x) = \int_{\mathbf{R}} p_t(y-x)u_0(y)dy + \int_{(0,t]\times\mathbf{R}} p_{t-r}(y-x)\sigma(u_r(y))W(drdy)$$

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• Does $u_t(x)$ have Markov property?

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Markov Property			

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• Does
$$u_t(x)$$
 have Markov property?

• If not,

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Markov Property			

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- Does $u_t(x)$ have Markov property?
- If not, this talk ends.

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- Does $u_t(x)$ have Markov property?
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- Thank you for your attention.
- BUT...

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Markov Property			

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$$u_{s+t}(x) = \int_{\mathbf{R}} p_t(y-x)u_s(y)dy + \int_{(0,t]\times\mathbf{R}} p_{t-r}(y-x)\sigma(u_{s+r}(y))W_s(drdy)$$

where $W_s(r, y) = W(s + r, y)$.

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Stochastic Heat Equations Intermittency	Dissipation	Thank 0

• Upper *p*th-moment Lyapunov exponent $\bar{\gamma}(p)$ of *u* at x_0 :

$$ar{\gamma}(p) := \limsup_{t o \infty} rac{1}{t} \log E(|u(t,x_0)|^p) ext{ for all } p \in (0,\infty).$$

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$$p
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 is nondecreasing.

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Stochastic Heat Equations Thank Intermittency Dissipation 0000 Lyapunov exponent and Intermittency

• Upper pth-moment Lyapunov exponent $\overline{\gamma}(p)$ of u at x_0 :

$$ar{\gamma}(p) := \limsup_{t o \infty} rac{1}{t} \log E(|u(t,x_0)|^p) ext{ for all } p \in (0,\infty).$$

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 is nondecreasing.

- Zeldovich, Molchanov, Ruzmajkin and Sokolov proposed a rigorous and constructive definition of asymptotic (as $t \to \infty$) intermittency.
- Full intermittency: if, regardless of the value of x_0 ,

$$p \rightarrow \frac{\bar{\gamma}(p)}{p}$$
 is strictly increasing for all $p \ge 2$.

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Remind that

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$$\ell \in (\bar{\gamma}(p)/p, \bar{\gamma}(q)/q)$$
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Stochastic Heat Equations			Intermittency 00000	Dissipation 000	Thank O
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Remind that

$$ar{\gamma}(p):=\limsup_{t o\infty}rac{1}{t}\log E(|u(t,x_0)|^p) ext{ for all } p\in(0,\infty).$$

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- Pick $\ell \in (\bar{\gamma}(p)/p, \bar{\gamma}(q)/q)$.
- Let $\mathcal{E}_t := \{ |u_t(x_0)| > \exp(t\ell) \}.$

Stochastic Heat Equations		Intermittency	Dissipation	Thank		
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Remind that

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- Pick $\ell \in (\bar{\gamma}(p)/p, \bar{\gamma}(q)/q).$
- Let $\mathcal{E}_t := \{ |u_t(x_0)| > \exp(t\ell) \}.$
- Chebychev's inequality gives

$$P(\mathcal{E}_t) \leq \exp(-pt\ell)\exp(\log E|u_t(x_0)|^p)$$

Stochastic I	Stochastic Heat Equations		Intermittency	Dissipation	Thank	
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Remind that

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Suppose that $ar\gamma(p)/p < ar\gamma(q)/q$ if p < q.

- Pick $\ell \in (\bar{\gamma}(p)/p, \bar{\gamma}(q)/q).$
- Let $\mathcal{E}_t := \{ |u_t(x_0)| > \exp(t\ell) \}.$
- Chebychev's inequality gives

$$\begin{array}{ll} \mathcal{P}(\mathcal{E}_t) &\leq & \exp(-pt\ell)\exp(\log E|u_t(x_0)|^p) \\ &= & \exp(-t[p\ell-t^{-1}\log E|u_t(x_0)|^p]) \to 0 \text{ as } t \to \infty. \end{array}$$

Stochastic I	Stochastic Heat Equations		Intermittency	Dissipation	Thank	
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• $E[|u_t(x_0)|^q 1_{\mathcal{E}^c_t}] \le e^{tq\ell} < e^{t\bar{\gamma}(q)} \approx E[|u_t(x_0)|^q]$

Stochastic H	Stochastic Heat Equations		Intermittency	Dissipation	Thank	
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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

Remind that $\mathcal{E}_t := \{ |u_t(x_0)| > \exp(t\ell) \}.$

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

Remind that $\mathcal{E}_t := \{ |u_t(x_0)| > \exp(t\ell) \}.$

• High peaks with small probability.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

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Remind that
$$\mathcal{E}_t := \{ |u_t(x_0)| > \exp(t\ell) \}.$$

- High peaks with small probability.
- High peaks contribute the moment.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

- High peaks with small probability.
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- The *p*-th moment $E|u_t(x)|^p$ is approximately $\exp(p^k t)$ where k > 1.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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SHE with domain R.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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SHE with domain **R**.

•
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.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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SHE with domain **R**.

- $E[|u_t(x)|^k] \le C^k \exp(Ck^3 t).$
- PAM, $E[|u_t(x)|^k] \approx C^k \exp(Ck^3 t)$ [Bertini-Cancrini, 1995].

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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- PAM, $E[|u_t(x)|^k] \approx C^k \exp(Ck^3 t)$ [Bertini-Cancrini, 1995].
- σ is bounded, $E|u_t(x)|^k = o(t^{k/2})$ [Foondun-Khoshnevisan, 2009].

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Chaotic Behaviors			

$$c < \limsup_{R \to \infty} rac{\log \sup_{|x| \le R} u_t(x)}{(\log R)^d} < C,$$

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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Chaotic Behaviors			

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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Chaotic Behaviors			

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Apply the Hoph-Cole transformation $u_t(x) := \exp(h_t(x))$. $\{h_t(x)\}$ satisfies the following KPZ equation:

$$rac{\partial}{\partial t}h_t(x) = rac{\partial^2}{\partial x^2}h_t(x) + (rac{\partial}{\partial x}h_t(x))^2 + rac{\partial^2}{\partial t\partial x}W(t,x).$$

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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The chaotic estimates for SHE give

$$a_t < \limsup_{|x| \to \infty} \frac{h_t(x)}{(\log |x|)^{2/3}} < A_t.$$

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

$$\frac{\partial}{\partial t}u_t(x) = \triangle u_t(x) + \lambda \sigma(u_t(x)) \frac{\partial^2}{\partial t \partial x} W(t,x), x \in [-1,1]$$

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

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with Neumann boundary condition and σ is approximately linear.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

$$\frac{\partial}{\partial t}u_t(x) = \triangle u_t(x) + \lambda \sigma(u_t(x)) \frac{\partial^2}{\partial t \partial x} W(t,x), x \in [-1,1]$$

with Neumann boundary condition and σ is approximately linear.

$$ct \leq \liminf_{\lambda \to \infty} \frac{\log \sqrt{\int_{-1}^{1} E|u_t(x)|^2 dx}}{\lambda^4} \leq \limsup_{\lambda \to \infty} \frac{\log \sqrt{\int_{-1}^{1} E|u_t(x)|^2 dx}}{\lambda^4} \leq Ct$$

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[K.W. Kim-Khoshnevisan, 2015].

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Moments			

$$\frac{\partial}{\partial t}u_t(x) = \triangle u_t(x) + \lambda \sigma(u_t(x)) \frac{\partial^2}{\partial t \partial x} W(t,x), x \in [-1,1]$$

with Neumann boundary condition and $\boldsymbol{\sigma}$ is approximately linear.

$$ct \leq \liminf_{\lambda \to \infty} \frac{\log \sqrt{\int_{-1}^{1} E|u_t(x)|^2 dx}}{\lambda^4} \leq \limsup_{\lambda \to \infty} \frac{\log \sqrt{\int_{-1}^{1} E|u_t(x)|^2 dx}}{\lambda^4} \leq Ct$$
[K.W. Kim-Khoshnevisan, 2015].

$$\liminf_{\lambda \to \infty} \frac{\log \log \inf_{x} E|u_{t}(x)|^{2}}{\log \lambda} = \limsup_{\lambda \to \infty} \frac{\log \log \sup_{x} E|u_{t}(x)|^{2}}{\log \lambda} = 4$$

[Foondun-Joseph, 2014].

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<u>Dissipation</u> Theorem. When λ goes to ∞

Consider

$$rac{\partial}{\partial t}u_t(x;\lambda)= riangle u_t(x;\lambda)+\lambda\sigma(u_t(x;\lambda))rac{\partial^2}{\partial t\partial x}W(t,x),\,x\in[-1,1]$$

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Dissipation Theorem. When λ goes to ∞

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with periodic boundary condition.

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Dissipation Theorem. When λ goes to ∞

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with periodic boundary condition. Recall that SHE with Neumann boundary condition $E|u_t(x;\lambda)|^2 \approx \exp(C\lambda^4 t)$ as $\lambda \to \infty$.

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Dissipation Theorem. When λ goes to ∞

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with periodic boundary condition. Recall that SHE with Neumann boundary condition $E|u_t(x;\lambda)|^2 \approx \exp(C\lambda^4 t)$ as $\lambda \to \infty$.

Theorem (Khoshnevisan-K.W. Kim-Mueller-S, 2016+)

For fixed t > 0

 $\sup_{x\in [-1,1]} u_t(x;\lambda) \to 0 \text{ in probability as } \lambda \to \infty.$

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Dissignation Theory			
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Dissipation Theorem. When $t \to \infty$

$E|u_t(x;\lambda)|^2 \ge C \exp(C\lambda^4 t)$ [Foondun-E.Nualart, 2015].

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Dissipation Theorem. When $t \to \infty$

$E|u_t(x;\lambda)|^2 \ge C \exp(C\lambda^4 t)$ [Foondun-E.Nualart, 2015].

Theorem (Khoshnevisan-K.W. Kim-Mueller-S, 2016+)

For all $\lambda > 0$,

$$\begin{array}{ll} -\infty &< \displaystyle \liminf_{t \to \infty} \frac{1}{t} \log \inf_{x \in [-1,1]} u_t(x;\lambda) \\ &\leq \displaystyle \limsup_{t \to \infty} \frac{1}{t} \log \sup_{x \in [-1,1]} u_t(x;\lambda) < 0 \ a.s. \end{array}$$

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Dissipation Theorem, Hohenberg-Swift

$$\begin{split} \frac{\partial}{\partial t}\psi_t(x;\lambda) &= \Delta\psi_t(x;\lambda) + \psi_t(x;\lambda) - \psi_t^3(x;\lambda) \\ &+ \lambda\sigma(\psi_t(x;\lambda)) \frac{\partial^2}{\partial t \partial x} W(t,x), x \in [-1,1]. \end{split}$$

with periodic boundary condition.

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Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Dissipation Theorem, Hohenberg-Swift

$$egin{aligned} &rac{\partial}{\partial t}\psi_t(x;\lambda) &= & riangle\psi_t(x;\lambda) + \psi_t(x;\lambda) - \psi_t^3(x;\lambda) \ &+ \lambda\sigma(\psi_t(x;\lambda))rac{\partial^2}{\partial t\partial x}W(t,x), x\in [-1,1]. \end{aligned}$$

with periodic boundary condition.

Theorem (Khoshnevisan-K.W. Kim-Mueller-S, 2016+)

There exists non-random constants 0 < $\lambda_1 < \infty$ such that: whenever $\lambda > \lambda_1$,

$$\limsup_{t\to\infty}\frac{1}{t}\log\sup_{x\in[-1,1]}\psi_t(x,\lambda)<0 \ a.s.$$

Stochastic Heat Equations	Intermittency	Dissipation	Thank
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Thank you

