Stochastic Hamiltonian Flows with Singular Coefficients

Xicheng ZHANG *Wuhan University, China*, E-mail: XichengZhang@gmail.com

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Abstract: In this work we study the following stochastic Hamiltonian system in R^{2d} (a second order stochastic differential equation),

$$d\dot{X}_t = b(X_t, \dot{X}_t)dt + \sigma(X_t, \dot{X}_t)dW_t, \quad (X_0, \dot{X}_0) = (x, v) \in R^{2d},$$

where $b(x,v) : R^{2d} \to R^d$ and $\sigma(x,v) : R^{2d} \to R^d \otimes R^d$ are two Borel measurable functions. We show that if σ is bounded and uniformly non-degenerate, and $b \in H_p^{2/3,0}$ and $\nabla\sigma \in L^p$ for some $p > 2(2d+1)$, where $H_p^{\alpha,\beta}$ is the Bessel potential space with differentiability indices α in x and β in v, then the above stochastic equation admits a unique strong solution so that $(x,v) \mapsto Z_t(x,v) := (X_t, \dot{X}_t)(x,v)$ forms a stochastic homeomorphism flow, and $(x,v) \mapsto Z_t(x,v)$ is weakly differentiable with $\text{ess.sup}_{x,v} E\left(\sup_{t\in[0,T]} |\nabla Z_t(x,v)|^q\right) < \infty$ for all $q \geq 1$ and $T \geq 0$. Moreover, we also show the uniqueness of probability measure-valued solutions for kinetic Fokker-Planck equations with rough coefficients by showing the well-posedness of the associated martingale problem and using the superposition principle established by Figalli [3] and Trevisan [5].

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