

# On the Martingale Problem and Feller and Strong Feller Properties for Weakly Coupled Lévy Type Operators

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**Abstract:** This work considers the martingale problem for a class of weakly coupled Lévy type operators. It is shown that under some mild conditions, the martingale problem is well-posed and therefore uniquely determines a strong Markov process  $(X, \Lambda)$ . The process  $(X, \Lambda)$ , called a regime-switching jump diffusion with Lévy type jumps, is further shown to possess Feller and strong Feller properties via the coupling method.

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