Heat Kernels of Non-Symmetric Jump Processes: beyond the Stable Case

Panki Kim *Seoul National University, Korea*
Renming SONG *University of Illinois, USA*, E-mail: rsong@illinois.edu
Zoran Vondracek *University of Zagreb, Croatia*

KEY WORDS: Heat kernel estimates, subordinate Brownian motion, symmetric Lévy process, non-symmetric operator, non-symmetric Markov process

MATHEMATICAL SUBJECT CLASSIFICATION: 60J35, 60J75

Abstract: Let J be the Lévy density of a symmetric Lévy process in $\mathbf{R^d}$ with its Lévy exponent satisfying a weak lower scaling condition at infinity. Consider the non-symmetric and non-local operator

$$\mathcal{L}^\kappa f(x) := \lim_{\epsilon \downarrow 0} \int_{\{z \in \mathbf{R^d}: |\mathbf{z}| > \epsilon\}} (f(x+z) - f(z)) \kappa(x,z) J(z)\, dz\,,$$

where $\kappa(x,z)$ is a Borel measurable function on $\mathbf{R^d} \times \mathbf{R^d}$ satisfying $0 < \kappa_0 \le \kappa(x,z) \le \kappa_1$, $\kappa(x,z) = \kappa(x,-z)$ and $|\kappa(x,z) - \kappa(y,z)| \le \kappa_2 |x-y|^\beta$ for some $\beta \in (0,1)$. We construct the heat kernel $p^\kappa(t,x,y)$ of $\mathcal{L}^\kappa$, establish its upper bound as well as its fractional derivative and gradient estimates. Under an additional weak upper scaling condition at infinity, we also establish a lower bound for the heat kernel p^κ.