

Regularity and Strict Positivity of Densities for Stochastic Fractional Heat Equation

Le Chen *University of Kansas, USA*

Yaozhong HU *University of Kansas, USA*, E-mail: yhu@ku.edu

David Nualart *University of Kansas, USA*

Abstract: In this paper, by using Malliavin calculus, we prove that the solution to a semilinear stochastic (fractional) heat equation with measure-valued initial data has a smooth joint density at multiple points. This is achieved by proving that the solutions to a related stochastic partial differential equation have negative moments of all orders. We also prove that the density is strictly positive in the interior of the support of the law, where we allow both measure-valued initial data and unbounded diffusion coefficient. One aim of this study is to cover the *parabolic Anderson model*.

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