Spatial Asymptotics for the Parabolic Anderson Models with Generalized Time-Space Gaussian Noise

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KEY WORDS: Generalized Gaussian field, white noise, fractional noise, Brownian motion, parabolic Anderson model, Feynman-Kac representation

MATHEMATICAL SUBJECT CLASSIFICATION: 60J65, 60K37,60K40, 60G55, 60F10

Abstract: Partially motivated by the work by Conus el, this work is concerned with the precise spatial asymptotic behavior for the parabolic Anderson equation

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\Delta u(t,x) + V(t,x)u(t,x) \\ \\ u(0,x) = u_0(x) \end{cases}$$

where the homogeneous generalized Gaussian noise $V(t,x)$ is, among other forms, white or fractional white in time and space. Associated with the Cole-Hopf solution to the KPZ equation, in particular, the precise asymptotic form

$$\lim_{R\to\infty} (\log R)^{-2/3} \log \max_{|x|\le R} u(t,x) = \frac{3}{4}\sqrt[3]{\frac{2t}{3}} \qquad a.s.$$

is obtained for the parabolic Anderson model $\partial_t u = \frac{1}{2}\partial^2_{xx} u + \dot{W} u$ with the $(1+1)$-white noise $\dot{W}(t,x)$.

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