

# Beijing-London-Swansea Workshop on Stochastic Analysis and Applications

Apr. 1 - 5, 2016

No. 1124 Lecture room on 11th floor, New Library Building (后主楼)  
Beijing Normal University

**Organizers:** Markus Riedle (King's College, London)

Feng-Yu Wang (Beijing Normal University/Swansea University)

Chenggui Yuan (Swansea University)

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	Apr. 1	Apr. 2	Apr. 3	Apr. 4	Apr. 5
<b>Chairman</b>	<b>Mu-Fa Chen</b>	<b>Boguslaw Zegarliniski</b>	<b>Markus Riedle</b>	<b>Tusheng Zhang</b>	<b>Feng-Yu Wang</b>
<b>09:00-09:45</b>	Boguslaw Zegarliniski	Take Picture	Zenghu Li	Eugene Lytvynov	Andrew Neate
<b>09:45-10:30</b>	Jiang-Lun Wu	Huizhong Zhao	Dmytro Finkelshtein	Yuhui Zhang	Chenggui Yuan
<b>10:30-11:00</b>	Tea break				
<b>11:00-11:45</b>	Jing-Hai Shao	Chunrong Feng	Chunge Liang	Dejun Luo	
<b>12:00-12:30</b>	Lunch				
<b>Chairman</b>	<b>Chenggui Yuan</b>	<b>Zenghu Li</b>	<b>Jiang-Lun Wu</b>	<b>Huazhong Zhao</b>	
<b>15:00-15:45</b>	Tusheng Zhang	Markus Riedle	Xiangdong Li	Teemu Pennanen	
<b>15:45-16:30</b>	Kristian Evans	Aleksandar Mijatovic	Mei Zhang	Wenming Hong	
<b>16:30-17:00</b>	Tea break				
<b>17:00-17:45</b>	Hui He	Yonghua Mao	Jianhai Bao	Daquan Jiang	
<b>18:00-18:30</b>	Supper	Banquet	Supper	Supper	

## April 1

09:00–09:45 Boguslaw Zegarlinski (Imperial College)

*Long time behaviour of infinite dimensional (nonlinear) dissipative systems*

09:45–10:30 Jiang-Lun Wu (Swansea University)

*Some recent results for stochastic scalar conservation laws with boundary conditions*

10:30–11:00 Tea break

11:00–11:45 Jinghai Shao (Beijing Normal University)

*Ergodicity of regime-switching diffusion processes in Wasserstein distance*

15:00–15:45 Tusheng Zhang (Manchester University)

*Quasilinear parabolic stochastic partial differential equations*

15:45–16:30 Kristian Evans (Swansea University)

*Feller semigroups on  $C_\infty(\mathbb{R}^n \times \mathbb{Z}^m)$  generated by pseudo-differential operators*

16:30–17:00 Tea break

17:00–17:45 Hui He (Beijing Normal University)

*From Galton-Watson trees to Levy trees: scaling limits and tree-valued processes*

## April 2

09:00–09:45 Take picture

09:45–10:30 Huaizhong Zhao (Loughborough University)

*Ergodicity of random periodic processes and periodic measures*

10:30–11:00 Tea break

- 11:00–11:45 Chunrong Feng (Loughborough University)  
*random periodic solutions of SDEs*
- 15:00–15:45 Markus Riedle (King’s College)  
*Stochastic integration with respect to cylindrical Levy processes*
- 15:45–16:30 Aleksandar Mijatovic (King’s College)  
*A weak multilevel monte carlo scheme for Levy processes*
- 16:30–17:00 Tea break
- 17:00–17:45 Mei Zhang (Beijing Normal University)  
*Large deviation for supercritical branching processes with immigration*

## April 3

- 09:00–09:45 Zenghu Li (Beijing Normal University)  
*Branching processes and stochastic equations*
- 09:45–10:30 Dmitri Finkelshtein (Swansea University)  
*Kinetic equations for stochastic dynamics of complex systems: derivation, analysis and ‘refinement’*
- 10:30–11:00 Tea break
- 11:00–11:45 Gechun Liang (King’s College)  
*Ergodic BSDE representation of forward performance processes*
- 15:00–15:45 Xiang dong Li (Chinese Academy of Science)  
*W-entropy formula on super Ricci flow and optimal transportation on manifolds, from perelman to Lott-Villani*
- 15:45–16:30 Yuhui Zhang (Beijing Normal University)  
*Unified treatment for some problems of single birth processes*
- 16:30–17:00 Tea break
- 17:00–17:45 Jianhai Bao (Central South University of China)  
*Strong limit theorems for two-time-scale SPDEs*

## April 4

09:00–09:45 Eugene Lytvynov (Swansea University)

*Gauge-invariant quasi-free states on the algebra of the anyon commutation relations and their renormalized particle density*

09:45–10:30 Yonghua Mao (Beijing Normal University)

*Adding vorticity matrix to the reversible Markov chain*

10:30–11:00 Tea break

11:00–11:45 Dejun Luo (Chinese Academy of Science)

*A class of stochastic differential equations with Osgood and Sobolev coefficients*

15:00–15:45 Teemu Pennanen (King's College, London)

15:45–16:30 Wenming Hong (Beijing Normal University)

*Scaling limit theorems for the  $\kappa$ -transient random walk in random environment*

16:30–17:00 Tea break

17:00–17:45 Daquan Jiang (Peking University)

*Cycle symmetry and circulation fluctuations of some Markov processes*

## April 5

09:00–09:45 Andrew Neate (Swansea University)

*Stochastic mechanics and the semiclassical Kepler/Coulomb problem*

09:45–10:30 Chenggui Yuan (Swansea University)

*Approximation of Invariant Measures for Regime-Switching Diffusions*

## STRONG LIMIT THEOREMS FOR TWO-TIME-SCALE SPDES

**Jianhai BAO** *Central South University of China, PRC*, E-mail: jianhaibao@csu.edu.cn

**Abstract:** This talk focuses on SPDEs under two-time-scale formulation. Distinct from the work in the existing literature, the systems are driven by  $\alpha$ -stable processes with  $\alpha \in (1, 2)$ . In addition, the SPDEs are either modulated by a continuous time Markov chain or have an additional fast jump component. The inclusion of the Markov chain is for the needs of treating random environment, whereas the addition of the fast jump process enables the consideration of discontinuity in the sample paths of the fast processes. Assuming either a fast changing Markov switching or an additional fast-varying jump process, this work aims to obtain the averaging principles for such systems. There are several distinct difficulties. First, the noise is not square integrable. Second, in our setup, for the underlying SPDE, there is only a unique mild solution and as a result, there is no Itô's formula that can be used. Moreover, another new aspect is the addition of the fast regime switching and the addition of the fast varying jump processes in the formulation, which enlarges the applicability of the underlying systems. To overcome these difficulties, a semigroup approach is taken. Under suitable conditions, it is proved that the  $p$ th moment convergence takes place with  $p \in (1, 2)$  uniformly, which is stronger than the usual weak convergence approaches. In addition, certain error bounds are also obtained.

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## FELLER SEMIGROUPS ON $C_\infty(\mathbb{R}^n \times \mathbb{Z}^m)$ GENERATED BY PSEUDO-DIFFERENTIAL OPERATORS

**Kristian P. EVANS** *Swansea University, UK*, E-mail: K.Evans@swansea.ac.uk  
**Niels JACOB** *Swansea University, UK*  
**Chenglin Shen** *Swansea University, UK*

**Abstract:** In this paper we construct some Feller semigroups, hence Feller processes, with state space  $\mathbb{R}^n \times \mathbb{Z}^m$  starting with pseudo-differential operators having symbols defined on  $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{Z}^m \times \mathbb{T}^m$ .

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## RANDOM PERIODIC SOLUTIONS OF SDES

**Chunrong FENG** *Loughborough University, UK*, E-mail: C.Feng@lboro.ac.uk

**Abstract:** In my talk, I will discuss the existence of random periodic solutions (r.p.s) for SDEs with additive and linear noise. We identify these as the solutions of coupled forward-backward infinite horizon stochastic integral equations in general cases. Then we use the Wiener-Sobolev compact embedding and Schauder's fixed point theorem to prove the existence of a solution of the integral equations. Finally, I will talk a specific example and prove that it has a r.p.s with minimum positive period. This is a joint work with Huaizhong Zhao.

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## KINETIC EQUATIONS FOR STOCHASTIC DYNAMICS OF COMPLEX SYSTEMS: DERIVATION, ANALYSIS AND 'REFINEMENT'

**Dmitri FINKELSHTEIN** *Swansea University, UK*, E-mail:  
D.L.Finkelshtein@swansea.ac.uk

**Abstract:** We describe the so-called statistical approach for the study of stochastic dynamics of complex systems in the continuum that allows deriving of the hierarchical equations for the correlations of all orders. We show how to rigorously reduce an appearing chain of linear equations (which reflect the microscopic behaviour of the systems) to a unique non-linear kinetic equation using mesoscopic scalings; the latter equation describes approximately the density of the rescaled system. We consider results about properties of solutions to such equations, in particular, travelling wave solutions and the front propagation. We present an approach that gives the next order of the approximation for the rescaled density.

## FROM GALTON-WATSON TREES TO LEVY TREES: SCALING LIMITS AND TREE-VALUED PROCESSES.

**Hui HE** *Beijing Normal University, Beijing*, E-mail: hehui@bnu.edu.cn

**Abstract:** Galton-Watson trees and Levy trees characterize genealogy structures of Galton-Watson processes and continuous state branching processes, respectively. In this talk, we first show that how Levy trees arise as scaling limits of Galton-Watson trees. Then we study some recent works on pruning trees and induced tree-valued processes.

## SCALING LIMIT THEOREMS FOR THE $\kappa$ -TRANSIENT RANDOM WALK IN RANDOM ENVIRONMENT

**Wenming HONG** *Beijing Normal University, PRC*, E-mail: wmhong@bnu.edu.cn

**Abstract:** Starting from a  $\kappa$ -transient RWRE (Kesten et al. (1975)), we will construct a sequence of the  $\kappa$ -transient RWREs and prove it convergence to the diffusion process with a  $\kappa/2$ -drifted Brownian potential (environment) by proper scaling. To this end, we obtain a counterpart convergence for the  $\kappa$ -transient random walk in non-random environment. (This is a joint work with Hui Yang)

## CYCLE SYMMETRY AND CIRCULATION FLUCTUATIONS OF SOME MARKOV PROCESSES

**Daquan JIANG** *Peking University, PRC*, E-mail: jiangdq@math.pku.edu.cn

**Abstract:**

Markov chains are widely used to model various stochastic systems in physics, chemistry, biology, etc. The trajectories of a recurrent Markov chain constantly form various cycles. For a family of cycles passing through the same set of states, we discover that the distributions of the forming times of these cycles, respectively conditioned on that the corresponding cycle is formed earlier than the others, are exactly the same. This cycle symmetry can be regarded as a generalization of the Haldane relation for reversible enzyme kinetics. We then find that this



cycle symmetry leads to the large deviation principle for the sample circulations along these cycles, in which the rate function has a non-obvious symmetry. This symmetry implies the Gallavotti-Cohen type fluctuation theorem for the sample net circulations. We also obtain other fluctuation theorems in non-equilibrium statistical physics for sample circulations.

Owing to a newly discovered symmetry of 1-dimensional Brownian motion, which says that it is invariant under quasi-time-reversal transformation, similar results hold for diffusion processes on the circle.

## W-ENTROPY FORMULA ON SUPER RICCI FLOW AND OPTIMAL TRANSPORTATION ON MANIFOLDS, FROM PERELMAN TO LOTT-VILLANI

Xiangdong LI, *Chinese Academy of Science, PRC*, E-mail: xdli@amt.ac.cn

**Abstract:** To complete Hamilton's program for the proof of the Poincaré conjecture, G. Perelman interpreted the Ricci flow as the gradient flow of the so-called F-entropy functional, and proved the W-entropy formula along the associated conjugate heat equation. Inspired by Perelman's work, we prove the W-entropy formula for the heat equation of the time dependent Witten Laplacian on manifolds equipped with super Ricci flow. We then prove the W-entropy formula for the transport equation and the Hamilton-Jacobi equation on manifolds. Our work recaptures Lott-Villani's theorem on the displacement convexity of the Boltzmann entropy on the Wasserstein space over Riemannian manifolds with non-negative Ricci curvature, an important result in the topic of the optimal transportation problem on manifolds. To better understand the above two results, we introduce the Langevin deformation of flows on the Wasserstein space, which interpolates the heat equation on manifolds and the geodesic flow on the cotangent bundle over the Wasserstein space, and can be considered as the potential flow of the compressible Euler equation with damping on Riemannian manifolds. The W-entropy formula will be extended to the deformation flows and the rigidity model is proposed. Joint work with Songzi Li (Fudan University and University Paul Sabatier).

## BRANCHING PROCESSES AND STOCHASTIC EQUATIONS

Zenghu LI *Beijing Normal University, PRC*, E-mail: lizh@bnu.edu.cn

**Abstract:** We present a number of stochastic integral equations in the theory of branching processes and random trees. We also explain how those stochastic equations can be used in the study the structural properties of the models.

## ERGODIC BSDE REPRESENTATION OF FORWARD PERFORMANCE PROCESSES

Gechun LIANG *King's College, UK*, E-mail: gechun.liang@kcl.ac.uk

**Abstract:** Ergodic BSDE representation of forward performance processes. Abstract: Forward performance processes were introduced by Musiela and Zariphopoulou, which complement the

classical expected utility paradigm where the utility is a deterministic function chosen at maturity, and there is little flexibility to incorporate updating of risk preferences, rolling horizons, learning and other realistic "forward in nature" features. Forward performance processes alleviate these shortcomings and offer a construction of a genuinely dynamic mechanism for evaluating the performance of investment strategies as the market evolves. In this talk, we show how to representant forward performance processes in terms of ergodic BSDE, and their connection with ergodic stochastic control.

## A CLASS OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH OSGOOD AND SOBOLEV COEFFICIENTS

Dejun LUO *Chinese Academy of Science, PRC*, E-mail: luodj@amss.ac.cn

**Abstract:** We consider the Ito stochastic differential equations (abbreviated as SDE) with degenerated and rough coefficients. More precisely, based on a unified treatment for SDEs under the Osgood and Sobolev type conditions, we show that, under suitable assumptions, the Ito SDE generates a unique stochastic flow of measurable maps.

## GAUGE-INVARIANT QUASI-FREE STATES ON THE ALGEBRA OF THE ANYON COMMUTATION RELATIONS AND THEIR RENORMALIZED PARTICLE DENSITY

Eugene LYTVYNOV *Swansea University, UK*, E-mail: E.Lytvynov@swansea.ac.uk

**Abstract:** Let  $X = \mathbb{R}^2$  and let  $q \in \mathbb{C}$ ,  $|q| = 1$ . For  $x = (x^1, x^2)$  and  $y = (y^1, y^2)$  from  $X^2$ , we define a function  $Q(x, y)$  to be equal to  $q$  if  $x^1 < y^1$ , to  $\bar{q}$  if  $x^1 > y^1$ , and to  $\Re q$  if  $x^1 = y^1$ . Let  $\partial_x^+$ ,  $\partial_x^-$  ( $x \in X$ ) be operator-valued distributions such that  $\partial_x^+$  is the adjoint of  $\partial_x^-$ . We say that  $\partial_x^+$ ,  $\partial_x^-$  satisfy the anyon commutation relations (ACR) if  $\partial_x^+ \partial_y^+ = Q(y, x) \partial_y^+ \partial_x^+$  for  $x \neq y$  and  $\partial_x^- \partial_y^+ = \delta(x - y) + Q(x, y) \partial_y^+ \partial_x^-$  for  $(x, y) \in X^2$ . In particular, for  $q = 1$ , the ACR become the canonical commutation relations and for  $q = -1$ , the ACR become the canonical anticommutation relations. We define the ACR algebra as the algebra generated by operator-valued integrals of  $\partial_x^+$ ,  $\partial_x^-$ . We construct a class of gauge-invariant quasi-free states on the ACR algebra. Each state from this class is completely determined by a positive self-adjoint operator  $T$  on the real space  $L^2(X, dx)$  which commutes with any operator of multiplication by a bounded function  $\psi(x^1)$ . In the case  $\Re q < 0$ , the operator  $T$  additionally satisfies  $0 \leq T \leq -1/\Re q$ . Further, for  $T = \kappa^2 \mathbf{1}$  ( $\kappa > 0$ ), we discuss the corresponding particle density  $\rho(x) := \partial_x^+ \partial_x^-$ . For  $\Re q \in (0, 1]$ , using a renormalization, we rigorously define a vacuum state on the commutative algebra generated by operator-valued integrals of  $\rho(x)$ . This state is given by a negative binomial point process. A scaling limit of these states as  $\kappa \rightarrow \infty$  gives the gamma random measure, depending on parameter  $\Re q$ .

## ADDING VORTICITY MATRIX TO THE REVERSIBLE MARKOV CHAIN

Yonghua MAO *Beijing Normal University, PRC*, E-mail: maoyh@bnu.edu.cn

**Abstract:** For a non-reversible finite Markov chain, by adding a vorticity matrix to the reversible transition probability matrix, we show that the commute time and average hitting time are smaller than that of the original reversible one. In particular, we give an affirmative answer to a conjecture in the book of Aldous and Fill.

## A WEAK MULTILEVEL MONTE CARLO SCHEME FOR LEVY PROCESSES

Aleksandar Mijatovic *King's College, UK*, E-mail: a.mijatovic@imperial.ac.uk

**Abstract:** We describe a novel weak multilevel approximation scheme for Levy processes. The scheme is based on the state space discretisation, via a continuous-time Markov chain approximation, of the driving Levy process and is particularly well suited if the multidimensional driver is given by a Levy copula. The multilevel version of the scheme is genuinely weak as it does not require strong convergence to control the level variances. The analysis of the level variances rests upon a new coupling between the approximating Markov chains of the consecutive levels, which is defined via a coupling of the corresponding Poisson Point Processes and is easy to simulate.

## STOCHASTIC MECHANICS AND THE SEMICLASSICAL KEPLER/COULOMB PROBLEM

Andrew Neate *Swansea University, UK*, E-mail: A.D.Neate@swansea.ac.uk

**Abstract:** We consider the semiclassical behaviour of a family of coherent states for the Coulomb potential which are concentrated on an ellipse with the aim of deriving Keplerian motion on the ellipse in the semiclassical limit. This is done within the framework of Nelson's stochastic mechanics which associates a diffusion process to each quantum state. We are led to a suitable semiclassical Nelson diffusion process which can be viewed as a solution for a stochastic perturbation of the Coulomb problem. We consider its relation to the classical constants of the motion (angular momentum, Lenz-Runge vector, energy) and consider the behaviour of an asymptotic series expansion for the semiclassical Nelson diffusion process.

## STOCHASTIC INTEGRATION WITH RESPECT TO CYLINDRICAL LEVY PROCESSES

Markus RIEDLE *King's College, UK*, E-mail: markus.riedle@kcl.ac.uk

**Abstract:** Cylindrical Levy processes are a natural generalisation of cylindrical Wiener processes and Gaussian white noise. However, since a cylindrical Levy process does not enjoy a cylindrical version of the semi-martingale decomposition, one cannot apply one of the standard approaches to define stochastic integrals with respect to cylindrical Levy processes. In this talk, we will introduce a completely novel approach to stochastic integration. In this approach the integrator is not decomposed into a martingale and a bounded variation process. As a consequence, the sequence of stochastic integrals for simple integrands can only be considered as a sequence in the space  $L^0$  of Hilbert space valued random variables. Convergence is established by tightness arguments utilising an approach called decoupled tangent sequences.

## ERGODICITY OF REGIME-SWITCHING DIFFUSION PROCESSES IN WASSERSTEIN DISTANCE

Jinghai SHAO *Beijing Normal University, PRC*, E-mail: shaojh@bnu.edu.cn

**Abstract:** In this talk, we provide some criteria for the ergodicity of regime-switching diffusion processes in Wasserstein distance. These criteria are given by using separately the M-matrix theory, Perron-Frobenius theorem and the estimate of principle eigenvalue. The state-dependent and state-independent regime-switching diffusion processes with switching in a finite set or a countable state space are all studied. The cost function we used to define the Wasserstein distance is not necessarily bounded. To deal with the countable state space, we put forward a new finite partition method, which studies the desired property via transforming the switching in a countable set to a switching in a finite set.

## SOME RECENT RESULTS FOR STOCHASTIC SCALAR CONSERVATION LAWS WITH BOUNDARY CONDITIONS

Jiang-Lun WU *Swansea University, UK*, E-mail: J.L.Wu@swansea.ac.uk

**Abstract:** In this talk we will review recent results for stochastic scalar conservation laws on bounded domains. We start with the well-posedness theory for stochastic scalar conservation laws with boundary conditions. We then discuss various type of solutions to the boundary value problems. Finally, we will give a positive answer to an open problem posed by Bauzet, Vallet and Wittbold in “The Dirichlet problem for a conservation law with a multiplicative stochastic perturbation”, *J. Funct. Anal.* 266 (2014) 2503-2545.

## APPROXIMATION OF INVARIANT MEASURES FOR REGIME-SWITCHING DIFFUSIONS

Chenggui YUAN *Swansea University, UK*, E-mail: C.Yuan@swansea.ac.uk

**Abstract:** In this talk, we are concerned with long-time behavior of Euler-Maruyama schemes associated with regime-switching diffusion processes. We will talk about existence and uniqueness of numerical invariant measures are addressed (i) for regime-switching diffusion processes with finite state spaces by the Perron-Frobenius theorem if the “averaging condition” holds, and, for the case of reversible Markov chain, via the principal eigenvalue approach provided that the principal eigenvalue is positive; (ii) for regime-switching diffusion processes with countable state spaces by means of a finite partition method and an M-Matrix theory. We also reveal that numerical invariant measures converge in the Wasserstein metric to the underlying ones.

## LONG TIME BEHAVIOUR OF INFINITE DIMENSIONAL (NONLINEAR) DISSIPATIVE SYSTEMS

Boguslaw ZEGARLINSKI *Imperial College, UK*, E-mail: b.zegarlinski@imperial.ac.uk

**Abstract:** This will be mostly about applications of log-Sobolev inequality to solving nonlinear problems in infinite dimensions.

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## LARGE DEVIATION FOR SUPERCRITICAL BRANCHING PROCESSES WITH IMMIGRATION

Mei ZHANG *Beijing Normal University, PRC*, E-mail: meizhang@bnu.edu.cn

**Abstract:** In this paper, we study the large deviation for a supercritical branching process with immigration controlled by a sequence of non-negative integer-valued independently identical distributed random variables, improving the previous results for non-immigration processes. We rely heavily on the detail description and limit property of the generating function of immigration processes.

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## QUASILINEAR PARABOLIC STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

Tusheng ZHANG *Manchester University, UK*, E-mail: Tusheng.Zhang@manchester.ac.uk

**Abstract:** We present a direct approach to existence and uniqueness of strong (in the probabilistic sense) and weak (in the PDE sense) solutions to quasilinear stochastic partial differential equations, which are neither monotone nor locally monotone. The proof of uniqueness is very elementary, based on a new method of applying Itô's formula for the  $L^1$ -norm. The proof of existence relies on a recent regularity result.

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## UNIFIED TREATMENT FOR SOME PROBLEMS OF SINGLE BIRTH PROCESSES

Yuhui ZHANG *Beijing Normal University, PRC*, E-mail: zhangyh@bnu.edu.cn

**Abstract:** In this talk, we survey the unified treatment for various criteria on classical problems (including uniqueness, recurrence, ergodicity, exponential ergodicity, strong ergodicity, as well as extinction probability etc.) for single birth processes, which is based on a new explicit representation of the solution to the Poisson equation. Meanwhile, a new representation of stationary distribution for ergodic single death processes and the mean occupation time for transient single birth processes are presented.

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## ERGODICITY OF RANDOM PERIODIC PROCESSES AND PERIODIC MEASURES

Huaizhong ZHAO *Loughborough University, UK*, E-mail: H.Zhao@lboro.ac.uk

**Abstract:** An ergodic theorem and a mean ergodic theorem in the random periodic regime on a Polish space are proved. In the Markovian random dynamical systems case, the idea of Poincare sections is introduced and under mixing assumption of the discrete time semigroup at multiple integrals of the period, the ergodicity of the periodic measure is obtained. The distinction between random periodic and stationary regimes is characterised by the spectral structure of the infinitesimal generators of the Markov semigroups. It is asserted that infinitesimal generator has only multiples of the quotient of  $2\pi$  and the minimum period as its simple eigenvalues on the imaginary axis if and only if the minimum period of the periodic measure is positive. The generator has only one simple eigenvalue 0 on the imaginary axis if and only if it is in the mixing stationary case. The latter agrees with what the classical Koopman-von Neumann theorem suggests. We also prove that the spectral gap of the semigroup on Poincare sections gives the exponential convergence of the mean of transition probability to the mean of the periodic measure over one period and therefore the periodic measure is ergodic.

## Participants: (in order of the family name)

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