Hypercontractivity, Compactness, and Exponential Ergodicity for Functional Stochastic Differential Equations

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Abstract: An explicit sufficient condition on the hypercontractivity is derived for the Markov semigroup associated to a class of functional stochastic differential equations. Consequently, the semigroup P_t converges exponentially to its unique invariant probability measure μ in both $L^2(\mu)$ and the totally variational norm, and it is compact in $L^2(\mu)$ for large $t > 0$. This provides a natural class of non-symmetric Markov semigroups which are compact for large time but non-compact for small time. A semi-linear model which may not satisfy this sufficient condition is also investigated.