Derivative for the intersection local time of fractional Brownian Motions

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Abstract: Let $B^{H_i}=\{B_t^{H_i},\, t\geq 0\}$, $i=1,2$ be two independent fractional Brownian motions on $\mathbb{R}$ with respective indices $H_i\in(0,1)$ and $H_1\leq H_2$. In this paper, we consider their intersection local time $\{\ell_t(a),t\geq 0,a\in\mathbb{R}\}$. We show that $\ell_t(a)$ is differentiable in the spatial variable a and we introduce the so-called *hybrid quadratic covariation* $[f(B^{H_1}-B^{H_2}),B^{H_1}]^{(HC)}$. When $H_1<\frac{1}{2}$, we construct a Banach space $\mathscr{H}$ of measurable functions such that the quadratic covariation exists in $L^2(\Omega)$ for all $f\in\mathscr{H}$, and the Bouleau-Yor type identity

$$[f(B^{H_1}-B^{H_2}),B^{H_1}]_t^{(HC)}=-\int_{\mathbb{R}}f(a)\ell_t(da)$$

holds. When $H_1\geq\frac{1}{2}$, we show that the quadratic covariation exists also in $L^2(\Omega)$ for some Hölder functions f and the above Bouleau-Yor type identity holds.