Exponential Contractivity in the L^p-Wasserstein Distance for Stochastic Differential Equations with Jumps

Dejun LUO *Academy of Mathematics and Systems Science, PRC*
Jian WANG *Fujian Normal University, PRC*, E-mail: jianwang@fjnu.edu.cn

Abstract: By adopting the coupling by reflection and the coupling of marching soldiers for a class of stochastic differential equations with jumps, we establish the exponential contractivity of the associated semigroups $(P_t)_{t\geq 0}$ with respect to the standard L^p-Wasserstein distance for all $p \in [1, \infty)$. In particular, consider the following stochastic differential equation

$$dX_t = dZ_t + b(X_t)\, dt,$$

where $(Z_t)_{t\geq 0}$ is symmetric α-stable process on R^d with $\alpha \in (1, 2]$. We show that if the drift term b satisfies that for any $x, y \in R^d$,

$$\langle b(x) - b(y), x - y\rangle \leq \begin{cases} K_1|x-y|^2, & |x-y| \leq L; \\ -K_2|x-y|^\theta, & |x-y| > L \end{cases}$$

holds with some positive constants K_1, K_2, $L > 0$ and $\theta \geq 2$, then there is a constant $\lambda > 0$ such that for all $p \in [1, \infty)$, $t > 0$ and $x, y \in R^d$ with $|x - y| \leq 1$,

$$W_p(\delta_x P_t, \delta_y P_t) \leq C(p, \theta)e^{-\lambda t/p}|x-y|^{1/p};$$

for all $p \in [1, \infty)$, $t > 0$ and $x, y \in R^d$ with $|x - y| > 1$,

$$W_p(\delta_x P_t, \delta_y P_t) \leq C(p, \theta)e^{-\lambda t/p} \begin{cases} |x-y|, & \theta = 2; \\ |x-y| \wedge \frac{1}{t\wedge 1}, & \theta > 2. \end{cases}$$