Variance reduction for diffusions

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Abstract: The most common way to sample from a probability distribution is to use Monte- Carlo methods. For distributions on a continuous state space, one can find diffusions with the target distribution as equilibrium measure, so that the state of the diffusion after a long time provides a good sample from the desired distribution. There exist many diffusions with a common equilibrium, and one would naturally like to choose those that make the convergence to equilibrium faster. One way to do this is to consider reversible diffusion, and add to it an anti-symmetric drift which preserves the invariant measure. It has been proven that, in general, the irreversible algorithm performs better than the reversible one, in that the spectral gap is larger. In the present work, asymptotic variance is used as the criterion to compare these algorithms. We first provide a general comparison result, and then apply it to the specific cases of a diffusion on d-dimensional Euclidean space, or on a compact Riemannian manifold. We prove that, in general, adding an anti-symmetric drift to a reversible diffusion reduces the asymptotic variance. We also provide some extensions of this result concerning strict inequality, the worst-case analysis, and the behavior of the asymptotic variance when the drift goes to infinity.