Density convergence for some nonlinear Gaussian stationary sequences

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Abstract: Consider a Gaussian stationary sequence with unit variance $X = \{X_k; k = 0, 1, 2, \cdots\}$ and a weighted sum of the form $V_n = n^{-1/2} \sum_{k=0}^{n-1} f(X_k)$, where f designates a finite sum of Hermite polynomials. Under some mild assumption involving the causal representation of X we show that the distributions of the random variables V_n have smooth densities ρ_n. Assume that the central limit theorem holds for V_n. Then we prove that the uniform convergence of the density of V_n towards the standard Gaussian density also holds true. Namely,

$$\lim_{n\to\infty} \sup_{x\in\mathbf{R}} |\rho_n(x) - \phi(x)| = 0\,,$$

where $\phi(x) = (2\pi)^{-1/2} e^{-x^2/2}$ is the normal density. The rate of convergence is also obtained. This is a joint work with Samy Tindel, David Nualart and Fangjun Xu.