Central Limit Theorem of Diffusion Processes with a Small Parameter in Discontinuous Media

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Abstract: For the system of $d-$dimesnional stochastic differential equation, $d \geq 2$,

$$dX_t^\epsilon = b(X_t^\epsilon)dt + \epsilon dW_t, \qquad t \in [0,T]$$
$$X_0^\epsilon = x \in H \subseteq R^d,$$

where $b(x) = (b_1(x), ..., b_d(x))$ is a bounded smooth vector field except along the hyperplane $H = \{x \in R^d, x_1 = 0\}$ but satisfies the stability condition in the sense that there exist positive constants δ and c such that $b_1(x) \leq -c$ if $x_1 \in (0, \delta)$ and $b_1(x) \geq c$ if $x_1 \in (-\delta, 0)$, we shall prove that the central limit theorem holds for $X^\epsilon(t)$. To be precise, we shall show that there exists a deterministic function $\phi(\cdot) \in C([0,T], R^d)$ such that the process $\frac{1}{\epsilon}(X^\epsilon(\cdot) - \phi(\cdot))$ converges to an Ornstein-Uhlenbeck proess in probability thus in distribution in $C([0,T], R^d)$ as $\epsilon \to 0$.