Pruning of CRT subtrees

Hui HE (何　辉)

Beijing Normal University

July 10, 2013@E'meishan

Motivations

- Characterization of super-critical Lévy tree:

  Duquesne and Winkel (2007):

  CRT as an almost sure limit of supercritical Galton-Watson trees.

  Abraham and Delmas (2012):

  Change of measure.

- Scaling limits of tree-valued processes:

  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

In this talk, we will only consider Brownian trees.

Motivations

- Characterization of super-critical Lévy tree:
  Duquesne and Winkel (2007):
  CRT as an almost sure limit of supercritical Galton-Watson trees.

  Abraham and Delmas (2012):
  Change of measure.

- Scaling limits of tree-valued processes:
  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

In this talk, we will only consider Brownian trees.

Motivations

- Characterization of super-critical Lévy tree:
  Duquesne and Winkel (2007):
  CRT as an almost sure limit of supercritical Galton-Watson trees.

  Abraham and Delmas (2012):
  Change of measure.

- Scaling limits of tree-valued processes:
  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

In this talk, we will only consider Brownian trees.

Motivations

- Characterization of super-critical Lévy tree:
  Duquesne and Winkel (2007):
  CRT as an almost sure limit of supercritical Galton-Watson trees.

  Abraham and Delmas (2012):
  Change of measure.

- Scaling limits of tree-valued processes:
  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

In this talk, we will only consider Brownian trees.

Motivations

- Characterization of super-critical Lévy tree:
  Duquesne and Winkel (2007):
  CRT as an almost sure limit of supercritical Galton-Watson trees.

  Abraham and Delmas (2012):
  Change of measure.

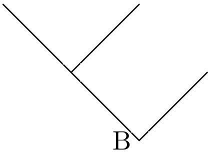
- Scaling limits of tree-valued processes:
  Discrete GW trees$\xrightarrow{d}$subtrees of CRTs$\xrightarrow{d}$CRTs.

In this talk, we will only consider Brownian trees.

Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

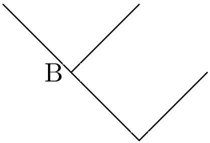
A bear searching a tree. B=Bear



Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

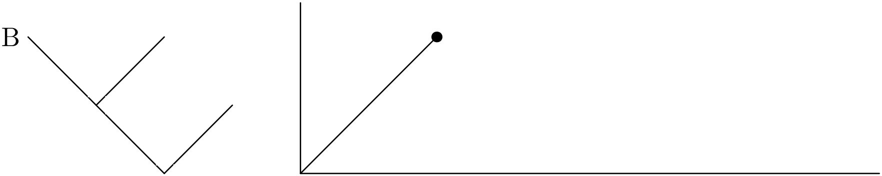
A bear searching a tree. B=Bear



Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

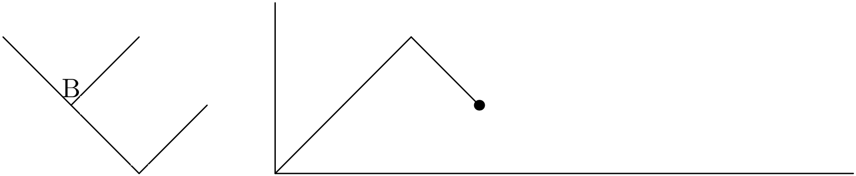
A bear searching a tree. B=Bear



Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

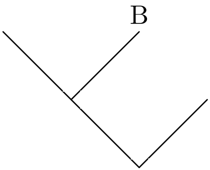
A bear searching a tree. B=Bear

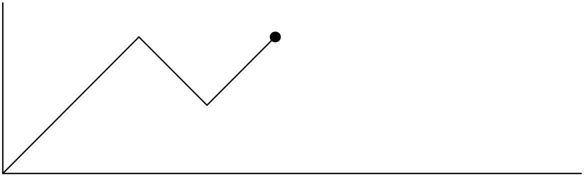


Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

A bear searching a tree. B=Bear

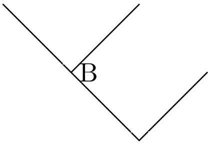


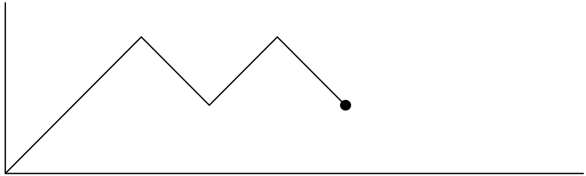


Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

A bear searching a tree. B=Bear

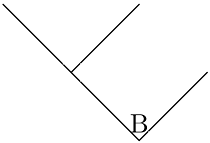


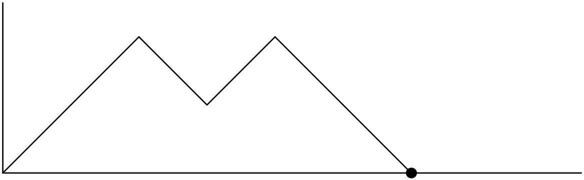


Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

A bear searching a tree. B=Bear

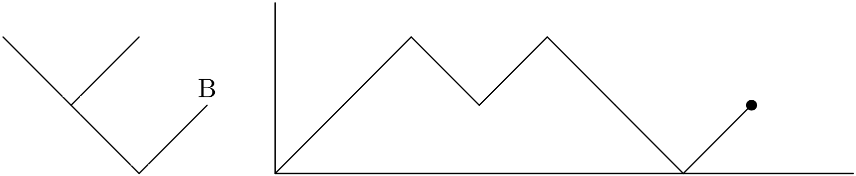




Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

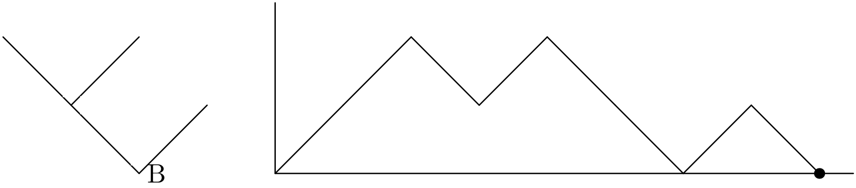
A bear searching a tree. B=Bear



Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

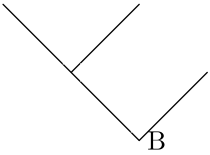
A bear searching a tree. B=Bear

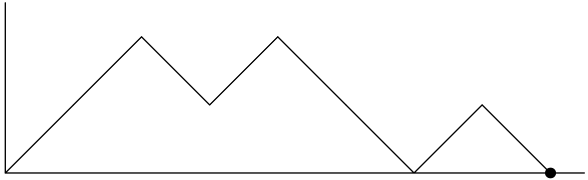


Real trees and contour functions

- Informally, real trees are metric spaces without loops, locally isometric to the real line.

A bear searching a tree. B=Bear





Brownian tree

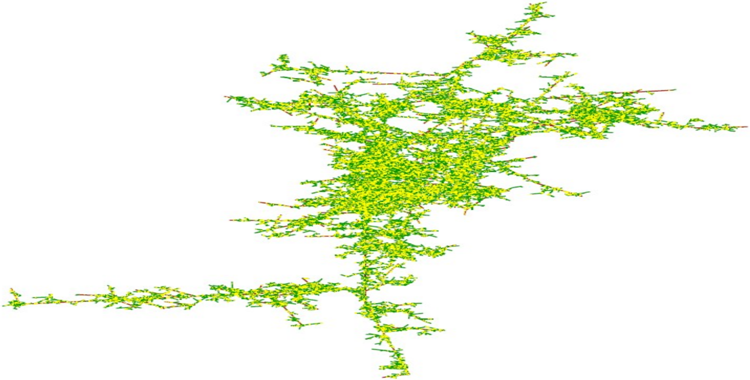
- A Brownian tree is a tree whose contour function is a reflected BM (Brownian excursion).
- Aldous (1990s):
  The continuum random trees I, II, III.
- Le Gall and Le Jan (97); Duquensne and Le Gall (2002):
  Lévy continuum random trees and genealogy of (sub)critical branching processes.

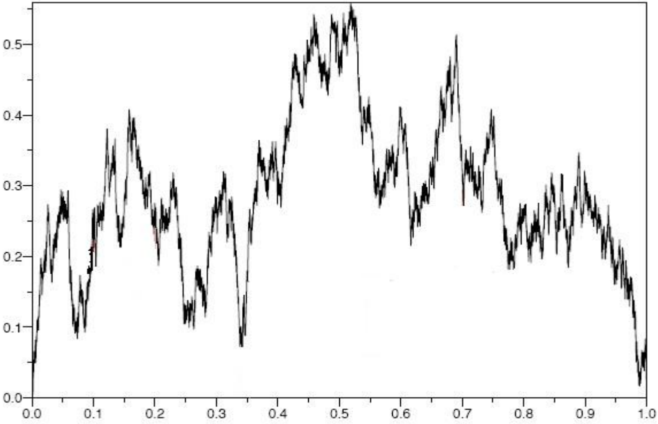
Brownian tree

- A Brownian tree is a tree whose contour function is a reflected BM (Brownian excursion).
- Aldous (1990s):
  The continuum random trees I, II, III.
- Le Gall and Le Jan (97); Duquensne and Le Gall (2002):
  Lévy continuum random trees and genealogy of (sub)critical branching processes.

Brownian tree

- A Brownian tree is a tree whose contour function is a reflected BM (Brownian excursion).
- Aldous (1990s):
  The continuum random trees I, II, III.
- Le Gall and Le Jan (97); Duquensne and Le Gall (2002):
  Lévy continuum random trees and genealogy of (sub)critical branching processes.




0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Motivations

- **Characterization of super-critical Lévy tree.**
- Scaling limits of tree-valued processes:
  pause Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

Motivations

- **Characterization of super-critical Lévy tree.**
- Scaling limits of tree-valued processes:
  pause Discrete GW trees$\xrightarrow{d}$subtrees of CRTs$\xrightarrow{d}$CRTs.

Super critical branching processes and trees

Question: How to construct trees for super-critical branching processes? Convergence and Characterization?

For a branching process Y, if

$$Y \text{ is } \begin{cases} \text{(sub)critical} & \text{then } \lim_{t\to\infty} Y_t = 0 \, a.s. \\ \text{super-critical} & \text{then } P\{\lim_{t\to\infty} Y_t = \infty\} > 0. \end{cases}$$

Super-critical trees will be infinite(non-compact) trees.

Super critical branching processes and trees

Question: How to construct trees for super-critical branching processes? Convergence and Characterization?

For a branching process Y, if

$$Y \text{ is } \begin{cases} \text{(sub)critical} & \text{then } \lim_{t\to\infty} Y_t = 0 \, a.s. \\ \text{super-critical} & \text{then } P\{\lim_{t\to\infty} Y_t = \infty\} > 0. \end{cases}$$

Super-critical trees will be infinite(non-compact) trees.

Super critical branching processes and trees

Question: How to construct trees for super-critical branching processes? Convergence and Characterization?

For a branching process Y, if

$$Y \text{ is } \begin{cases} \text{(sub)critical} & \text{then } \lim_{t\to\infty} Y_t = 0\, a.s. \\ \text{super-critical} & \text{then } P\{\lim_{t\to\infty} Y_t = \infty\} > 0. \end{cases}$$

Super-critical trees will be infinite(non-compact) trees.

Super critical branching processes and trees

Question: How to construct trees for super-critical branching processes? Convergence and Characterization?

For a branching process Y, if

$$Y \text{ is } \begin{cases} \text{(sub)critical} & \text{then } \lim_{t\to\infty} Y_t = 0 \, a.s. \\ \text{super-critical} & \text{then } P\{\lim_{t\to\infty} Y_t = \infty\} > 0. \end{cases}$$

Super-critical trees will be infinite(non-compact) trees.

Question: How to construct trees for super-critical branching processes? Convergence and Characterization?

For a branching process Y, if

$$Y \text{ is } \begin{cases} \text{(sub)critical} & \text{then } \lim_{t\to\infty} Y_t = 0 \, a.s. \\ \text{super-critical} & \text{then } P\{\lim_{t\to\infty} Y_t = \infty\} > 0. \end{cases}$$

Super-critical trees will be infinite(non-compact) trees.

Supercritical trees

Duquesne and Winkel (2007):

CRT as an almost sure limit of increasing Galton-Watson trees.

Abraham and Delmas (2012):

Connect to subcritical trees via change of meansure.

Main result: The limit tree in DW07 satisfies the Girsanov transformation introduced in AD12.

Supercritical trees

Duquesne and Winkel (2007):
CRT as an almost sure limit of increasing Galton-Watson trees.

Abraham and Delmas (2012):
Connect to subcritical trees via change of meansure.

Main result: The limit tree in DW07 satisfies the Girsanov transformation introduced in AD12.

Supercritical trees

Duquesne and Winkel (2007):

CRT as an almost sure limit of increasing Galton-Watson trees.

Abraham and Delmas (2012):

Connect to subcritical trees via change of meansure.

Main result: The limit tree in DW07 satisfies the Girsanov transformation introduced in AD12.

Basic ideas

- Abraham and Delmas' definition is the 'right' one;
- Constructing subtrees (Galton-Watson trees);
- Connect subtrees of super-critical trees to subtrees of subcritical trees via a similar change of measure;
- Law of subtrees is the same to the increasing tree-valued process define in Duquensne and Winkel (2007);
- Take limits.

Basic ideas

- Abraham and Delmas' definition is the 'right' one;
- Constructing subtrees (Galton-Watson trees);
- Connect subtrees of super-critical trees to subtrees of subcritical trees via a similar change of measure;
- Law of subtrees is the same to the increasing tree-valued process define in Duquensne and Winkel (2007);
- Take limits.

Basic ideas

- Abraham and Delmas' definition is the 'right' one;
- Constructing subtrees (Galton-Watson trees);
- Connect subtrees of super-critical trees to subtrees of subcritical trees via a similar change of measure;
- Law of subtrees is the same to the increasing tree-valued process define in Duquensne and Winkel (2007);
- Take limits.

Basic ideas

- Abraham and Delmas' definition is the 'right' one;
- Constructing subtrees (Galton-Watson trees);
- Connect subtrees of super-critical trees to subtrees of subcritical trees via a similar change of measure;
- Law of subtrees is the same to the increasing tree-valued process define in Duquensne and Winkel (2007);
- Take limits.

Basic ideas

- Abraham and Delmas' definition is the 'right' one;
- Constructing subtrees (Galton-Watson trees);
- Connect subtrees of super-critical trees to subtrees of subcritical trees via a similar change of measure;
- Law of subtrees is the same to the increasing tree-valued process define in Duquensne and Winkel (2007);
- Take limits.

Basic setting

- Denote by T_θ the trees with contour functions $X^\theta = B_t - 2\theta t - \inf_{s\le t}(B_s - 2\theta s)$. ($\theta > 0$.)
- By change of measure, Abraham and Delmas (2012) extends the definition of T_θ to $\theta < 0$.
- Denote by p_θ the canonical projection from support of contour functions onto T_θ.
- The mass measure on T_θ, denoted by $\mathbf{m}^\theta$, is the image measure on T_θ of the Lebesgue measure by p_θ (Concentrate on set of leaves).
- For $\theta < 0$, $\mathbf{m}^\theta$ can also be defined by Change of measure; see Abraham and Delmas (2012).

Basic setting

- Denote by T_θ the trees with contour functions $X^\theta = B_t - 2\theta t - \inf_{s\le t}(B_s - 2\theta s)$. ($\theta > 0$.)
- By change of measure, Abraham and Delmas (2012) extends the definition of T_θ to $\theta < 0$.
- Denote by p_θ the canonical projection from support of contour functions onto T_θ.
- The mass measure on T_θ, denoted by $\mathbf{m}^\theta$, is the image measure on T_θ of the Lebesgue measure by p_θ (Concentrate on set of leaves).
- For $\theta < 0$, $\mathbf{m}^\theta$ can also be defined by Change of measure; see Abraham and Delmas (2012).

Basic setting

- Denote by T_θ the trees with contour functions $X^\theta = B_t - 2\theta t - \inf_{s\leq t}(B_s - 2\theta s)$. ($\theta > 0$.)
- By change of measure, Abraham and Delmas (2012) extends the definition of T_θ to $\theta < 0$.
- Denote by p_θ the canonical projection from support of contour functions onto T_θ.
- The mass measure on T_θ, denoted by $\mathbf{m}^\theta$, is the image measure on T_θ of the Lebesgue measure by p_θ (Concentrate on set of leaves).
- For $\theta < 0$, $\mathbf{m}^\theta$ can also be defined by Change of measure; see Abraham and Delmas (2012).

Basic setting

- Denote by T_θ the trees with contour functions $X^\theta = B_t - 2\theta t - \inf_{s\le t}(B_s - 2\theta s)$. ($\theta > 0$.)
- By change of measure, Abraham and Delmas (2012) extends the definition of T_θ to $\theta < 0$.
- Denote by p_θ the canonical projection from support of contour functions onto T_θ.
- The mass measure on T_θ, denoted by $\mathbf{m}^\theta$, is the image measure on T_θ of the Lebesgue measure by p_θ (Concentrate on set of leaves).
- For $\theta < 0$, $\mathbf{m}^\theta$ can also be defined by Change of measure; see Abraham and Delmas (2012).

Basic setting

- Denote by T_θ the trees with contour functions $X^\theta = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$. ($\theta > 0$.)
- By change of measure, Abraham and Delmas (2012) extends the definition of T_θ to $\theta < 0$.
- Denote by p_θ the canonical projection from support of contour functions onto T_θ.
- The mass measure on T_θ, denoted by $\mathbf{m}^\theta$, is the image measure on T_θ of the Lebesgue measure by p_θ (Concentrate on set of leaves).
- For $\theta < 0$, $\mathbf{m}^\theta$ can also be defined by Change of measure; see Abraham and Delmas (2012).

Basic setting

- Denote by T_θ the trees with contour functions $X^\theta = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$. ($\theta > 0$.)
- By change of measure, Abraham and Delmas (2012) extends the definition of T_θ to $\theta < 0$.
- Denote by p_θ the canonical projection from support of contour functions onto T_θ.
- The mass measure on T_θ, denoted by $\mathbf{m}^\theta$, is the image measure on T_θ of the Lebesgue measure by p_θ (Concentrate on set of leaves).
- For $\theta < 0$, $\mathbf{m}^\theta$ can also be defined by Change of measure; see Abraham and Delmas (2012).

Subtree

Given a tree T_θ ($\theta \in \mathbb{R}$), consider a Poisson point measure:

$$P^\theta(dt, dx) = \sum_{i \in I^\theta} \delta_{(t_i, x_i)}$$

$R_+ \times T_\theta$ with intensity measure $dt\, \mathbf{m}^\theta(dx)$.

Define the subtree of T by

$$\tau(\theta, \lambda) = \bigcup \{[\![\emptyset, x_i]\!], i \in I^\theta \text{ and } t_i \leq \lambda\}, \qquad (1)$$

for $\lambda > 0$

Subtree

Given a tree T_θ ($\theta \in \mathbb{R}$), consider a Poisson point measure:

$$P^\theta(dt, dx) = \sum_{i \in I^\theta} \delta_{(t_i, x_i)}$$

$R_+ \times T_\theta$ with intensity measure $dt\, \mathbf{m}^\theta(dx)$.
Define the subtree of T by

$$\tau(\theta, \lambda) = \bigcup \{[\![\emptyset, x_i]\!], i \in I^\theta \text{ and } t_i \le \lambda\}, \tag{1}$$

for $\lambda > 0$

Law of subtrees

Define

$$\tau^{(a)}(\theta, T) = \{x \in T_\theta : d(\emptyset, x) \le a\}.$$

Theorem

Law of $\tau^{(a)}(\theta, \lambda)$ is absolutely continuous w.r.t. $\tau^{(a)}(-\theta, \lambda)$.

- The proof of the result is based on properties of Poisson random measure and Girsanov transformation for CRTs.

By results on distributions of Galton-Watson real trees and a result in Duquesne and Le Gall (2002), we immediately get

Corollary

$\tau(\theta, \lambda)$ is a Galton-Watson real tree.

When ψ is (sub)critical, the above result was proved by Duquesne and Le Gall (2002).

Law of subtrees

Define

$$\tau^{(a)}(\theta, T) = \{x \in T_\theta : d(\emptyset, x) \le a\}.$$

Theorem

Law of $\tau^{(a)}(\theta, \lambda)$ *is absolutely continuous w.r.t.* $\tau^{(a)}(-\theta, \lambda)$.

- The proof of the result is based on properties of Poisson random measure and Girsanov transformation for CRTs.

By results on distributions of Galton-Watson real trees and a result in Duquesne and Le Gall (2002), we immediately get

Corollary

$\tau(\theta, \lambda)$ *is a Galton-Watson real tree.*

When ψ is (sub)critical, the above result was proved by Duquesne and Le Gall (2002).

Law of subtrees

Define

$$\tau^{(a)}(\theta, T) = \{x \in T_\theta : d(\emptyset, x) \le a\}.$$

Theorem

Law of $\tau^{(a)}(\theta, \lambda)$ *is absolutely continuous w.r.t.* $\tau^{(a)}(-\theta, \lambda)$.

- The proof of the result is based on properties of Poisson random measure and Girsanov transformation for CRTs.

By results on distributions of Galton-Watson real trees and a result in Duquesne and Le Gall (2002), we immediately get

Corollary

$\tau(\theta, \lambda)$ *is a Galton-Watson real tree.*

When ψ is (sub)critical, the above result was proved by Duquesne and Le Gall (2002).

Law of subtrees

Define

$$\tau^{(a)}(\theta, T) = \{x \in T_\theta : d(\emptyset, x) \leq a\}.$$

Theorem

Law of $\tau^{(a)}(\theta, \lambda)$ *is absolutely continuous w.r.t.* $\tau^{(a)}(-\theta, \lambda)$.

- The proof of the result is based on properties of Poisson random measure and Girsanov transformation for CRTs.

By results on distributions of Galton-Watson real trees and a result in Duquesne and Le Gall (2002), we immediately get

Corollary

$\tau(\theta, \lambda)$ *is a Galton-Watson real tree.*

When ψ is (sub)critical, the above result was proved by Duquesne and Le Gall (2002).

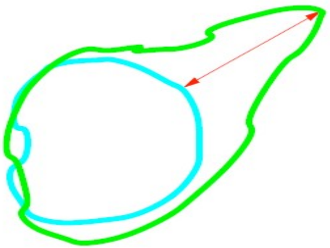
Tree space: Gromov-Hausdorff distance

Q: What is a random tree?(σ-algebra?)

Hausdorff distance: A, B, non-empty, closed subsets of a Polish metric space (X, d).

$$d_{\mathrm{H}}(A, B) = \inf\{\varepsilon > 0,\ A \subset B^{\varepsilon} \text{ and } B \subset A^{\varepsilon}\},$$

with $A^{\varepsilon} = \{x \in X, \inf_{y \in A} d(x, y) < \varepsilon\}$, the ε-halo set of A.

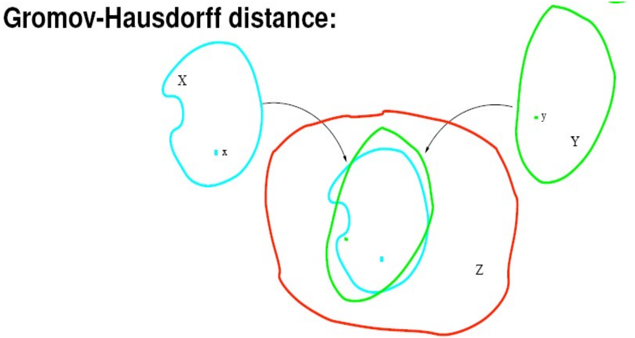


Gromov-Hausdorff distance:

Let $(X, d, \emptyset)$ and $(X', d', \emptyset')$ be two compact rooted metric spaces, and define:

$$d_{\rm GH}(X, X') = \inf_{\Phi, \Phi', Z} \left(d_{\rm H}^{Z}(\Phi(X), \Phi'(X')) + d^{Z}(\Phi(\emptyset), \Phi'(\emptyset')) \right),$$

where the infimum is taken over all isometric embeddings $\Phi : X \hookrightarrow Z$ and $\Phi' : X' \hookrightarrow Z$ into some common Polish metric space (Z, d^Z)



For X, X', locally compact rooted trees, define

$$d_{\mathrm{GH}}^{c}(X, X') = \int_0^\infty e^{-r}(1 \wedge d_{\mathrm{GH}}(X^{(r)}, X'^{(r)}))dr,$$

where $X^{(r)} = \{x \in X : d(\emptyset, x) \leq r\}$.

Let $\mathbb{T}$ be the set of (GH-isometry classes of) locally compact rooted trees.

Theorem

(Duquesne and Winkel (2007)) $(\mathbb{T}, d_{GH}^{c})$ is a Polish metric space.

For X, X', locally compact rooted trees, define

$$d^c_{\mathrm{GH}}(X, X') = \int_0^\infty e^{-r}(1 \wedge d_{\mathrm{GH}}(X^{(r)}, X'^{(r)}))dr,$$

where $X^{(r)} = \{x \in X : d(\emptyset, x) \leq r\}$.

Let $\mathbb{T}$ be the set of (GH-isometry classes of) locally compact rooted trees.

Theorem

(Duquesne and Winkel (2007)) $(\mathbb{T}, d^c_{GH})$ is a Polish metric space.

For X, X', locally compact rooted trees, define

$$d_{\mathrm{GH}}^{c}(X, X') = \int_0^{\infty} e^{-r}(1 \wedge d_{\mathrm{GH}}(X^{(r)}, X'^{(r)}))dr,$$

where $X^{(r)} = \{x \in X : d(\emptyset, x) \leq r\}$.

Let $\mathbb{T}$ be the set of (GH-isometry classes of) locally compact rooted trees.

Theorem

(Duquesne and Winkel (2007)) $(\mathbb{T}, d_{GH}^{c})$ is a Polish metric space.

Two references on metric geometry

- Gromov, M. (1999): Metric Structures for Riemannian and non-Riemannian Spaces. Progress in Mathematics.
- Burago, Y., Burago, D., Ivanov, S.(2001): A Course in Metric Geometry, vol. 33. AMS, Boston (Google)

Convergence from subtrees to CRT

Theorem

$$\lim_{\lambda \to +\infty} d^c_{GH}(T, \tau(\lambda)) = 0 \quad a.e. \tag{2}$$

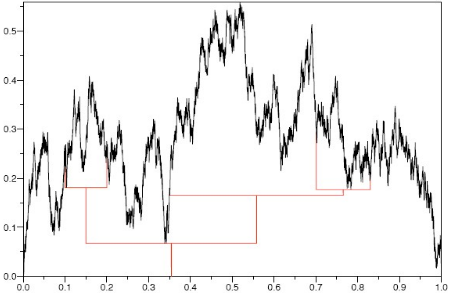
- The result recover the main result in Duquesne and Winkel (2007).
- This gives that the limit tree obtained in Duquesne and Winkel (2007) satisfies the definition given in Abraham and Delmas (2012).

- The proof is based on Girsanov transformation (**??**).
- We first prove that for ψ is (sub)critical

$$\lim_{\lambda \to +\infty} d^c_{\mathrm{GH}}(T, \tau(\lambda)) = 0 \quad ,$$

by approximating contour process by contour functions of $\tau(\lambda)$ and using the fact $d^c_{\mathrm{GH}}(T_f, T_g) \leq 6||f - g||$.

- Then by connections to subcritical trees, we get the desired result for supercritical case.



Motivations

- Characterization of super-critical Lévy tree.
- **Scaling limits of tree-valued processes.**
  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

Motivations

- Characterization of super-critical Lévy tree.
- **Scaling limits of tree-valued processes.**

  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

Motivations

- Characterization of super-critical Lévy tree.
- **Scaling limits of tree-valued processes.**
  Discrete GW trees $\xrightarrow{d}$ subtrees of CRTs $\xrightarrow{d}$ CRTs.

Poissonian raining on Brownian trees

Given a Brownian tree T

- At time t_i, there is a drop of sulfuric acid (硫酸) falling on the tree at $x_i \in T$.
- We cut the tree at x_i.
- $\{t_1 < t_2 < \cdots,\}$ is a Poisson process and $\{x_i, i = 1, 2, \cdots\}$ are uniformly distributed on T.
- $T(\theta)$ = remaining tree after time θ.
- $T(\theta)$ is tree whose contour function is X_t, where $X_t = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$; see Abraham and Delmas (2012).
- $\{T(\theta) : \theta \ge 0\}$ is a decreasing real tree-valued process.

Poissonian raining on Brownian trees

Given a Brownian tree T

- At time t_i, there is a drop of sulfuric acid (硫酸) falling on the tree at $x_i \in T$.
- We cut the tree at x_i.
- $\{t_1 < t_2 < \cdots ,\}$ is a Poisson process and $\{x_i, i = 1, 2, \cdots\}$ are uniformly distributed on T.
- $T(\theta)$ = remaining tree after time θ.
- $T(\theta)$ is tree whose contour function is X_t, where $X_t = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$; see Abraham and Delmas (2012).
- $\{T(\theta) : \theta \ge 0\}$ is a decreasing real tree-valued process.

Poissonian raining on Brownian trees

Given a Brownian tree T

- At time t_i, there is a drop of sulfuric acid (硫酸) falling on the tree at $x_i \in T$.
- We cut the tree at x_i.
- $\{t_1 < t_2 < \cdots,\}$ is a Poisson process and $\{x_i, i = 1, 2, \cdots\}$ are uniformly distributed on T.
- $T(\theta)$ = remaining tree after time θ.
- $T(\theta)$ is tree whose contour function is X_t, where $X_t = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$; see Abraham and Delmas (2012).
- $\{T(\theta) : \theta \ge 0\}$ is a decreasing real tree-valued process.

Poissonian raining on Brownian trees

Given a Brownian tree T

- At time t_i, there is a drop of sulfuric acid (硫酸) falling on the tree at $x_i \in T$.
- We cut the tree at x_i.
- $\{t_1 < t_2 < \cdots,\}$ is a Poisson process and $\{x_i, i = 1, 2, \cdots\}$ are uniformly distributed on T.
- $T(\theta)$ = remaining tree after time θ.
- $T(\theta)$ is tree whose contour function is X_t, where $X_t = B_t - 2\theta t - \inf_{s\le t}(B_s - 2\theta s)$; see Abraham and Delmas (2012).
- $\{T(\theta) : \theta \ge 0\}$ is a decreasing real tree-valued process.

Poissonian raining on Brownian trees

Given a Brownian tree T

- At time t_i, there is a drop of sulfuric acid (硫酸) falling on the tree at $x_i \in T$.
- We cut the tree at x_i.
- $\{t_1 < t_2 < \cdots ,\}$ is a Poisson process and $\{x_i, i = 1, 2, \cdots\}$ are uniformly distributed on T.
- $T(\theta)$ = remaining tree after time θ.
- $T(\theta)$ is tree whose contour function is X_t, where $X_t = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$; see Abraham and Delmas (2012).
- $\{T(\theta) : \theta \ge 0\}$ is a decreasing real tree-valued process.

Poissonian raining on Brownian trees

Given a Brownian tree T

- At time t_i, there is a drop of sulfuric acid (硫酸) falling on the tree at $x_i \in T$.
- We cut the tree at x_i.
- $\{t_1 < t_2 < \cdots,\}$ is a Poisson process and $\{x_i, i = 1, 2, \cdots\}$ are uniformly distributed on T.
- $T(\theta)$ = remaining tree after time θ.
- $T(\theta)$ is tree whose contour function is X_t, where $X_t = B_t - 2\theta t - \inf_{s \le t}(B_s - 2\theta s)$; see Abraham and Delmas (2012).
- $\{T(\theta) : \theta \ge 0\}$ is a decreasing real tree-valued process.

subtree processes

Define the subtree process

$$\tau_\theta(\lambda) = \tau(0, \lambda) \cap T_\theta.$$

Proposition

$\tau_\theta(\lambda)$ is a Galton-Watson real tree.

$$\lim_{\lambda\to\infty}\sup_{\theta\geq 0} d^c_{GH}(T_\theta, \tau_\theta(\lambda)) = 0 \quad a.e.$$

For further applications of above results; work in progress and see you on next workshop.

References

- R. Abraham, J.-F. Delmas (2012): *A continuum-tree-valued Markov process*, Annals of Probability.
- R. Abraham, J.-F. Delmas and H. He (2012+): *Pruning of CRT-sub-trees*, arXiv:1212.2765.
- T. Duquesne, M. Winkel (2007): *Growth of Lévy forest*, Probab. Theory Relat. Fields.

Thanks!