

HEAT KERNELS AND ANALYTICITY OF NON-SYMMETRIC LEVY DIFFUSION SEMIGROUPS

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Abstract:

Consider the following non-local and non-symmetric Lévy operator: for $\alpha \in (0, 2)$,

$$\mathcal{L}_\alpha^\kappa f(x) := \text{P.V.} \int_{\mathbb{R}^d} (f(x+z) - f(x)) \kappa(x, z) |z|^{-d-\alpha} dz,$$

where $\kappa(x, z) = \kappa(x, -z)$, $0 < \kappa_0 \leq \kappa(x, z) \leq \kappa_1$ and $|\kappa(x, z) - \kappa(y, z)| \leq \kappa_2 |x - y|^\beta$ for some $\beta \in (0, 1)$. Using Levi's method, we construct the heat kernel of $\mathcal{L}_\alpha^\kappa$, and prove the sharp upper bound, fractional derivative and gradient estimates of the heat kernel. Moreover, we also obtain the analyticity of the non-symmetric semigroup associated with $\mathcal{L}_\alpha^\kappa$ in L^p -spaces provided $p \in [1, \infty)$.