

# BROWNIAN MOTION AND THERMAL CAPACITY

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**Abstract:** Let  $W$  denote  $d$ -dimensional Brownian motion. We find an explicit formula for the essential supremum of Hausdorff dimension of  $W(E) \cap F$ , where  $E \subset (0, \infty)$  and  $F \subset \mathbf{R}^d$  are arbitrary nonrandom compact sets. Our formula is related intimately to the thermal capacity of Watson (1978). We prove also that when  $d \geq 2$ , our formula can be described in terms of the Hausdorff dimension of  $E \times F$ , where  $E \times F$  is viewed as a subspace of space time.

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