

SOME PROBLEMS ON RIEMANNIAN PATH AND LOOP SPACES

Bo WU *School of Mathematical Sciences, Fudan University, China*, E-mail: wubo@fudan.edu.cn

KEY WORDS: Dirichlet form, closability, quasi-regularity, path space, loop space, functional inequality

MATHEMATICAL SUBJECT CLASSIFICATION: 60H07

Abstract: In this talk we shall review some recent results on Riemannian path and loop spaces respectively. We construct a class of Quasi-regular Dirichlet form on path space without additional curvature conditions, and we also obtain a weighted log-Sobolev inequality with respect to the damped O-U Dirichlet form. In particular, Poincaré inequality can be derived under some unbounded curvature conditions. In addition, we establish the formula of integration by parts and functional inequalities on free loop space under some conditions.

Main References

- [1] B. K. Driver, M. Röckner(1992), Construction of diffusions on path and loop spaces of compact Riemannian manifolds, *C. R. Acad. Sci. Paris Séries*, **I 315**, 603–608
- [2] J.-U. Löbus(2004), A class of processes on the path space over a compact Riemannian manifold with unbounded diffusion, *Tran. Ame. Math. Soc.*, 1–17
- [3] F. Y. Wang, B. Wu(2008), Quasi-Regular Dirichlet Forms on Riemannian Path and Loop Spaces, *Forum Math.*, **20**, 1085–1096
- [4] S.- Z. Fang(1994), Un inégalité du type Poincaré sur un espace de chemins, *C. R. Acad. Sci. Paris Série*, **I 318**, 257–260
- [5] L. Gross(1991), Logarithmic Sobolev inequalities on loop groups. *J. Funct. Anal.*, **102**, 268 -313
- [6] S. Aida (2000), Logarithmic derivatives of heat kernels and logarithmic Sobolev inequalities with unbounded diffusion coefficients on loop spaces, *J. Funct. Anal.*, **174** 430-477
- [7] A. Eberle (2002), Absence of spectral gaps on a class of loop spaces. *J. Math. Pures. Appl.*, (**9**) **81**, 915-955
- [8] F. Z. Gong, Z. M. Ma(1998), The log-Sobolev inequality on loop space over a compact Riemannian manifold, *J. Funct. Anal.*, **157**, 599-623
- [9] X. M. Li, X. Chen, B. Wu(2010), A Poincaré inequality on loop spaces, *J. Funct. Anal.*, **259** 1421-1442
- [10] X. M. Li, X. Chen, B. Wu(2011), A concrete estimate for the weak Poincaré inequality on loop space, *Probab. Theory Relat. Fields*, **151**, 559-590