LAPLACIAN PERTURBED BY NON-LOCAL OPERATORS

Jieming WANG *School of Mathematics, Beijing Institute of Technology, PRC.* E-mail: wangjm@bit.edu.cn

Key words: Laplacian, perturbation, non-local operator, integral kernel, Lévy system, Feller semigroup, martingale problem

Abstract: Suppose that $d \geq 1$ and $0 < \beta < 2$. We establish the existence and uniqueness of the fundamental solution $q^b(t,x,y)$ to non-local operator $\mathcal{L}^b = \Delta + \mathcal{S}^b$, where

$$\mathcal{S}^b f(x) := \int_{\mathbb{R}^d} \left(f(x+z) - f(x) - \nabla f(x) \cdot z \mathbb{1}_{\{|z| \leq 1\}} \right) \frac{b(x,z)}{|z|^{d+\beta}} dz$$

and $b(x,z)$ is a bounded measurable function on $\mathbb{R}^d \times \mathbb{R}^d$ with $b(x,z) = b(x,-z)$ for $x,z \in \mathbb{R}^d$. We show that if $b(x,z) \geq 0$, then $q^b(t,x,y)$ is a strictly positive continuous function and it uniquely determines a conservative Feller process X^b, which has strong Feller property. The Feller process X^b is the unique solution to the martingale problem of $(\mathcal{L}^b, \mathcal{S}(\mathbb{R}^d))$, where $\mathcal{S}(\mathbb{R}^d)$ denotes the space of tempered functions on $\mathbb{R}^d$. Furthermore, sharp two-sided estimates on $q^b(t,x,y)$ are derived.