THE HEISENBERG INEQUALITY ON ABSTRACT WIENER SPACES

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Abstract: The Heisenberg inequality associated with the uncertainty principle is extended to an infinite dimensional abstract Wiener space (H, B) with an abstract Wiener measure p_1. For $\varphi \in L^2(p_1)$ and $T \in L(B, H)$, it is shown that

$$\Big[\int_B |Tx|_H^2 \, |\varphi(x)|^2 \, p_1(dx)\Big]\Big[\int_B |Tx|_H^2 \, |\mathcal{F}\varphi(x)|^2 \, p_1(dx)\Big] \geq ||T|_H||_{HS}^4 \, ||\varphi||_2^4,$$

where $\mathcal{F}\varphi$ is the Fourier-Wiener transform of φ. The conditions when the equality holds also discussed.

References

[1] Gross, L. (1965). Abstract Wiener spaces, *Proc. 5th Berkeley Symp. Math. Stat. Prob.*, **2**, 31-42.

[2] Gross, L. (1967). Potential theory on Hilbert spaces, *J. Funct. Anal.*, **1**, 123-181.

[3] Kuo, H.-H. (1974). Integration by parts for abstract Wiener measures, *Duke Math. J.* , **41**, 373-378.

[4] Kuo, H.-H. (1975). Gaussian Measure in Banach Spaces, *Lecture Notes in Math.*, **463**, Springer-Verlag.

[5] Lee, Y.J. (1991). Analytic version of test functionals, Fourier transform and a characterization of measures in white noise calculus, *J. Func. Anal.*, **100**, 359-380.

[6] Lee, Y.J. and A. Stan. An infinite dimensional Heisenberg uncertainty principle, *Taiwanese J. Math.*, **3(4)**, 529-538.