ASYMPTOTIC NORMALITY OF OCCUPATION TIME OF SINGULARLY PERTURBED DIFFUSION PROCESSES

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KEY WORDS: Central Limit Theorem, Singularly Perturbed Diffusions.

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60J60; Secondary 60F05,35C20.

Abstract: Let X_t^ϵ to be a diffusion process on the unit circle S^1 with generator $L^\epsilon = 1/\epsilon \cdot L_1 + L_2$ where $L_i(t,x) = b_i(t,x)\partial x + 1/2 \cdot a_i(t,x)\partial xx$ are two diffusion generators for $i = 1, 2$. Here, $b_i(t,x)$ and $a_i(t,x)\,(> 0)$ are taken to be smooth functions on S^1. Given a bounded measurable function $f(x)$ on S^1, an unscaled function of the occupation time of X_t^ϵ is defined as

$$Z^\epsilon(t,f) = \int_0^t (f(X^\epsilon(s)) - \int_0^1 f(y)p(s,y)dy)ds.$$

where for each $t, p(t,y)$ is the quasi-stationary distribution of $L_1(t,y)$. In this paper, we shall first show that the law of large numbers of $Z^\epsilon(t,f)$ holds, i.e., $\lim_{\epsilon\to 0} Z^\epsilon(t,f) = 0$ in L^2 and hence in probability. Let $n^\epsilon(t,f) = 1/\sqrt{\epsilon} \cdot Z^\epsilon(t,f)$. The second result is to show that $n^\epsilon(\cdot,f)$ converges to a Gaussian process $n(\cdot,f)$ as $\epsilon \to 0$ and we will explicitly compute the covariance function of $n(\cdot,f)$.