Convergence in Density of Some Nonlinear Gaussian Functionals

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Abstract: The central limit theorem states that If $X_1, \cdots, X_n, \cdots$ are iid then

$$F_n := \sqrt{n}\left(\frac{X_1 + \cdots + X_n}{n} - E(X_1)\right) \quad \text{converges in disribution to a normal random variable}.$$

This convergence also holds for some other random sequences F_n such as those given by multiple integrals with respect to Brownian motion ([2], [3]). We study the problem of when the random variable F_n has density $f_n(x), x \in R$ and when the densities $f_n(x)$ converges in L^p to the normal density $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$. We also applied our general results to least squares estimator for Ornstein-Uhlenbeck processes ([1]). The tool to use is Malliavin calculus. This is an ongoing joint work with Fei LU and David NUALART.

References

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