Weak Extinction Versus Global Exponential Growth of Total Mass for Super-diffusions Corresponding to the Operator $Lu + \beta u - ku^2$

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Abstract: Consider a superdiffusion X on $\mathbb{R}^d$ corresponding to the semilinear operator $Lu + \beta u - ku^2$, where L is a second order elliptic operator, $\beta(\cdot)$ is in the Kato class, and $k(\cdot) \geq 0$ is nonvanishing and bounded on compact subsets of $\mathbb{R}^d$. Let λ_∞ be the L^∞-growth bound of the semigroup $\{Q_t\}_{t\geq 0}$ corresponding to the Schrödinger operator $L + \beta$. If $\lambda_\infty \neq 0$, then we prove that, in some sense, the exponential growth/decay rate of $\langle 1, X_t\rangle$, the total mass of X_t, is λ_∞. We also describe the limiting behavior of $\exp(-\lambda_\infty t)\langle 1, X_t\rangle$ in these cases. This should be compared to the known result that the generalized principal eigenvalue λ_2 of the operator gives the rate of *local* growth when it is positive, and implies local extinction otherwise. It is easy to show that $\lambda_\infty \geq \lambda_2$, and we discuss cases when $\lambda_\infty > \lambda_2$ and when $\lambda_\infty = \lambda_2$.

When $\lambda_\infty = 0$, and under some conditions on β, we give a sufficient and necessary condition for the superdiffusion X to exhibit weak extinction. We show that the branching intensity k affects weak extinction; this should be compared to the known result that k does not affect weak *local* extinction (which only depends on the sign of λ_2, and which turns out to be equivalent to local extinction) of X.

References

[1] Chen, Z.-Q.(2002). Gaugeability and conditional Gaugeability, Trans. Amer. Math. Soc., **354**: 4639–4679.

[2] Chen, Z.-Q.(2011). Uniform Integrability of exponential martingales and spectral bounds of non-local Feynman-Kac semigroups. Preprint.

[3] Chen, Z.-Q. and Song, R.(2002). General gauge and conditional gauge theorems, Ann. Probab., **30**: 1313–1339.

[4] Engländer, J. and Kyprianou, A. E.(2004). Local extinction versus local exponential growth for spatial branching processes, Ann. Probab., **32**: 78–99.

[5] Engländer, J. and Pinsky, R. G. (1999). On the construction and support properties of measure-valued diffusions on $D \subset \mathbb{R}^d$ with spatially dependent branching, Ann. Probab., **27**: 684–730.

[6] Engländer, J. and Turaev, D.(2002). A scaling limit theorem for a class of superdiffusions, Ann. Probab., **30**: 286–722.

[7] Engländer, J. and Winter, A.(2006). Law of large numbers for a class of superdiffusions, Ann. I. H. Poincaré-PR, **42**: 171–185.

[8] Murata, M.(1984). Positive solutions and large time behaviour of Schrödinger semigroup, Simon's problem, J. Funct. Anal.,**56**: 300–310.

[9] Pinsky, R. G.(1996) Transience, recurrence and local extinction properties of the support for supercritical finite measure-valued diffusions. Ann. Probab., **24**: 237–267.

[10] Simon, B. (1981). Large time behaviuor of the L^p norm of Schrödinger semigroups, J. Funct. Anal., **40**: 66–83.

[11] Zhao, Z.(1992) Subcriticality and gaugeability of the Schrodinger operator, Trans. Amer. Math. Soc., **334**: 75–96.